

# **Synthesis of Time-modulated Linear Arrays for MIMO Applications through a PSO-based Strategy**

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## **Abstract**

This report analyzes the performance of a PSO-based strategy applied to the synthesis of harmonic multi-beam patterns for receiving multiple signals incoming on a time-modulated array from different angular directions. Different scenario configurations have been investigated and different swarm initializations have been taken into account in order to properly evaluate the capabilities of the proposed optimization procedure to maximize the signal-to-noise-plus-interference ratio (SINR).

## Introduction

The goal of this report is to present the effect of changing the parameters of the PSO. The cost function to be used in the simulations is defined as follows:

$$\begin{aligned}
 \Omega &= \eta_1 \Omega_1 - \eta_2 \Omega_2 - \eta_3 \Omega_3 \\
 \Omega_1 &= \sum_{h=-H}^H \left[ \frac{P_d^{(h)}}{P_d^{(h)} + P_u^{(h)} + P_n^{(h)}} \delta(h) \right] \\
 \Omega_2 &= \sum_{h=-H}^H \left[ \left( \frac{P_d^{max} - P_d^{(h)}}{P_d^{max}} \right)^2 \delta(h) \right] \\
 \Omega_3 &= \sum_{h=-H}^H \left[ \left( \frac{SLL^{(h)} - SLL_{ref}^{(h)}}{SLL_{peak}^{(h)} - SLL_{ref}^{(h)}} \right)^2 \Upsilon \left( SLL^{(h)} - SLL_{ref}^{(h)} \right) \delta(h) \right]
 \end{aligned}$$

**Cost Function**

where:

- $P_d^{(h)}$  is the power received in the  $h^{th}$  harmonic, from the signal desired in this harmonic.
- $P_u^{(h)} = \sum_{i=0}^{I-1} P_i^{(h)}$  is the sum of power received in the  $h^{th}$  harmonic from all signal sources but the desired signal in this harmonic.
- $P_n^{(h)} = \frac{1}{2} \underline{W}_{(h)}^H \Phi_n \underline{W}_{(h)}$  is the noise power captured in the  $h^{th}$  harmonic.
- $\underline{W}_{(h)}$  is a column vector of complex harmonic element weights, whose  $n^{th}$  element  $W_{(h)}[n]$  is given as:
  - $W_{(h)}[n] = A_n U_{hn} e^{j h w_p t}$
  - $A_n$  is complex static element weight.
  - $U_{hn}$  is the complex fourier coefficient of the time modulating function  $u_n(t)$ .
- $\underline{W}_{(h)}^H$  is the hermetian transpose of  $\underline{W}_{(h)}$ .
- $\Phi_n$  is the noise covariance matrix.
- $P_i^{(h)}$  is the power received from signal source  $i$ , in the  $h^{th}$  harmonic.
- $P_d^{max}$  is the maximum of all the desired signals  $P_d^{(h)}$ .
- $\delta(h) = \begin{cases} 1 & \text{if } h \text{ is included in the synthesis} \\ 0 & \text{otherwise} \end{cases}$
- $\Upsilon$  is the Heaviside function
- $\eta_1 \in [0, 1]$ ,  $\eta_2 \in [0, 1]$ ,  $\eta_3 \in [0, 1]$  are the weights of the components of the cost function.
- $SLL^{(h)}$  is the side lobe level of the  $h^{th}$  harmonic beam pattern.
- $SLL_{peak}^{(h)}$  is the peak of the pattern of the  $h^{th}$  harmonic beam.
- $SLL_{ref}^{(h)}$  is a reference level in the  $h^{th}$  harmonic beam pattern.

## Simulation Parameters

The following parameters are common to all simulations.

- Isotropic Array Elements:  $N = 20$
- Uniformly distributed along the  $z$  axis:  $x_n = 0, y_n = 0, z_n = \frac{n\lambda}{2}$
- Uniform amplitude and phase weighting of elements:  $A_n = 1$
- Initial Pulse width:  $\tau_n = 0.25$
- Initial Pulse shift:  $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- $\varphi_n$  phase of the array element weights
- $z_n$  is the  $z$  coordinate of the elements.
- $m$  is an integer chosen such that the constraint,  $0 \leq i_n^r \leq 1$ , is fulfilled.
- PSO Parameters
  - Number of Variables:  $X = 40$  ( $\tau_n, i_n^r, n = 1, 2, \dots, N$ )
  - Swarm Size: 40
  - Seed of Random Generator: 2500
- Signal and Noise parameters
  - Three Signals
  - Harmonic Index:  $h = 0, 1, 2$
  - Amplitude and phase for all Signal Sources:  $S_i = 1$
  - Noise Power:  $\varphi_n = -20dB$
  - Noise Covariance Matrix:  $\Phi_n = \varphi_n 1^N$

## TEST CASE 1 - $\theta_i = 90^\circ, 115.66^\circ, 150^\circ$

### TEST CASE 1.a

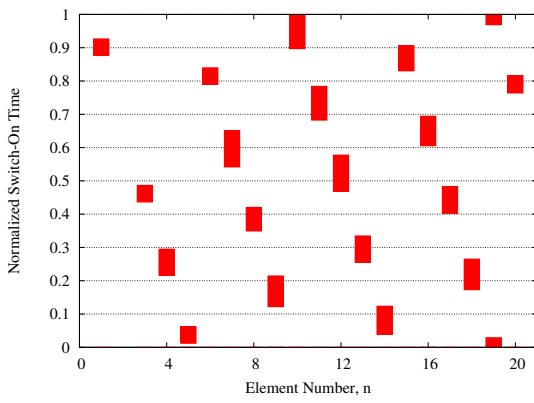
#### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

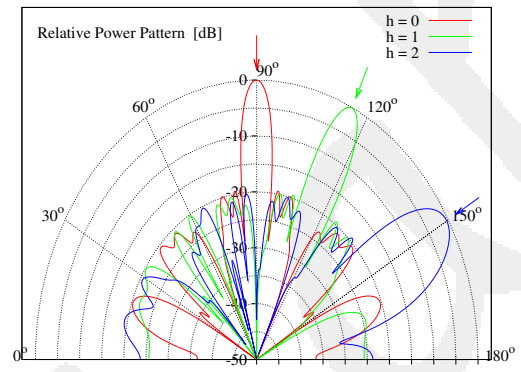
#### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 115.66^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$

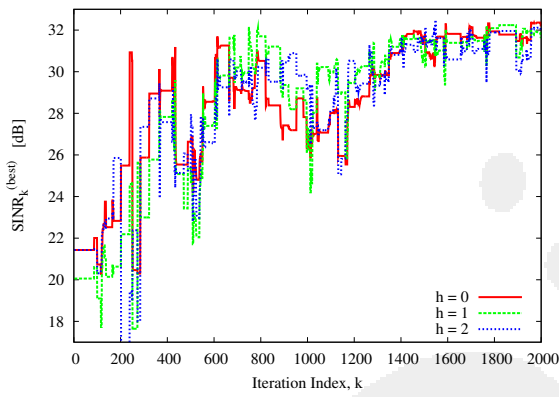
## Results



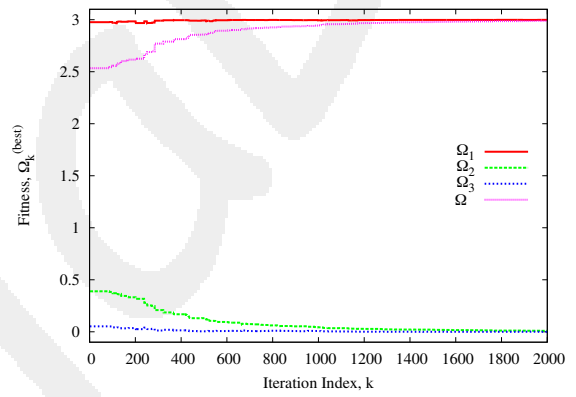
**Fig.1 - Pulse Sequence**



**Fig.2 - Pattern**



**Fig.3 - SINR**



**Fig.4 - Fitness**

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
<i>PSO</i>	32.225941	31.658371	31.959023	-19.495227	-19.444693	-19.383537	-0.105214	-0.426213

**Tab. 1 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).**

## TEST CASE 1.b

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 115.66^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$

## Results

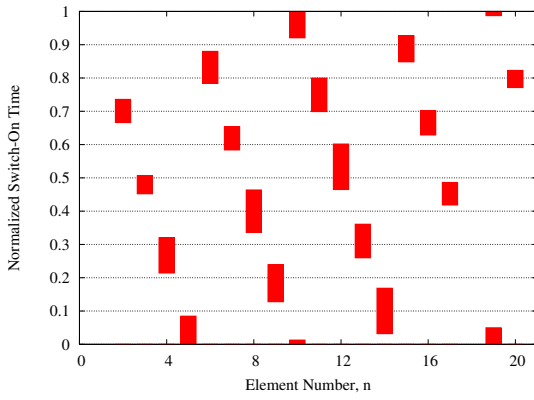


Fig.5 - Pulse Sequence

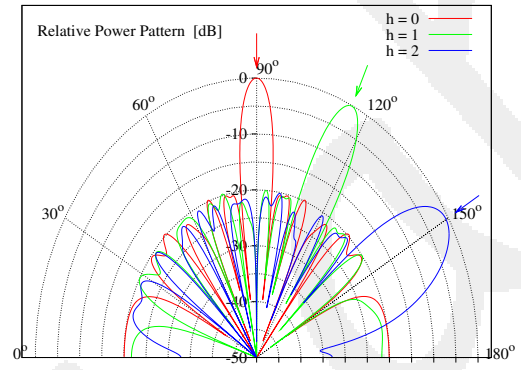


Fig.6 - Pattern

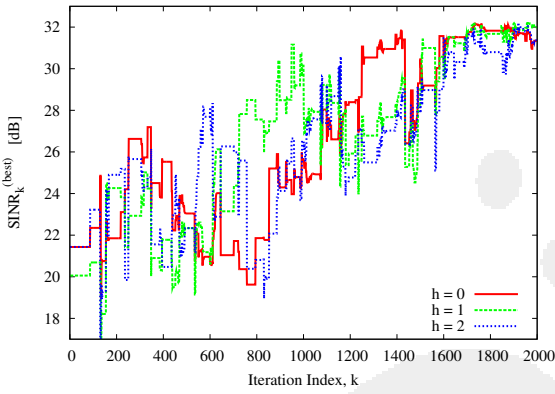


Fig.7 - SINR

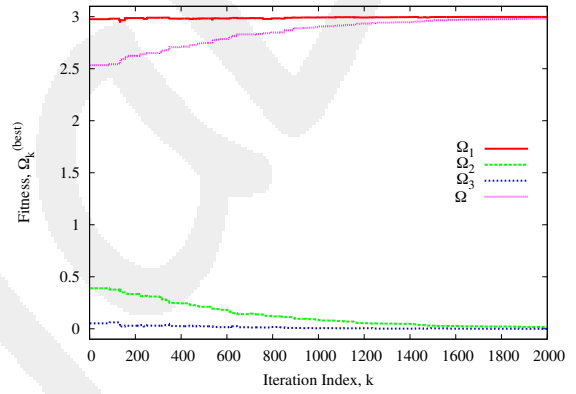


Fig.8 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	31.346038	31.993697	31.365908	-20.565234	-20.383721	-20.499021	-0.146270	-0.591391

Tab. 2 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 2 - $\theta_i = 90^\circ, 125^\circ, 150^\circ$

### TEST CASE 2.a

#### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

#### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$



## Results

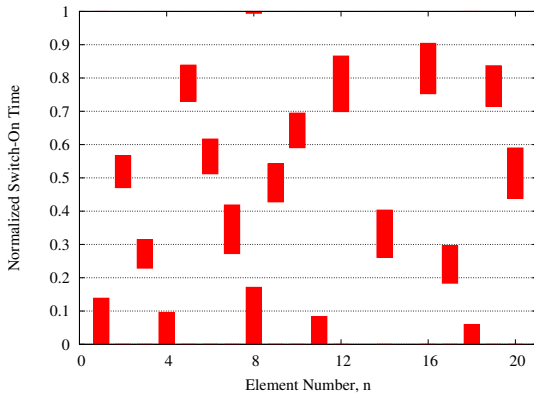


Fig.9 - Pulse Sequence

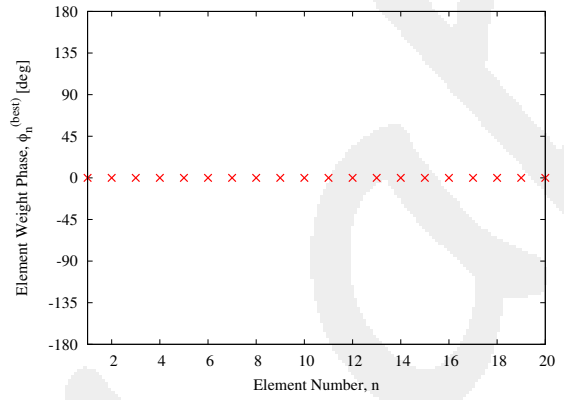


Fig.10 - Phases

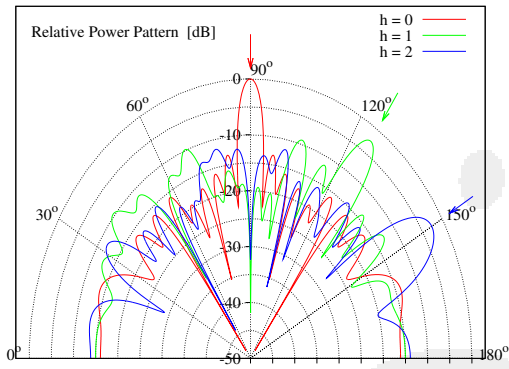


Fig.11 - Pattern

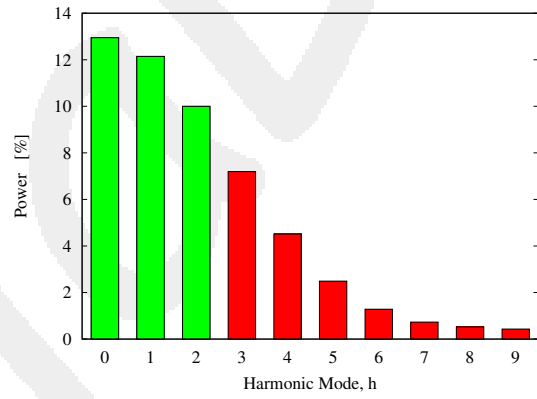


Fig.12 - Sideband Radiation

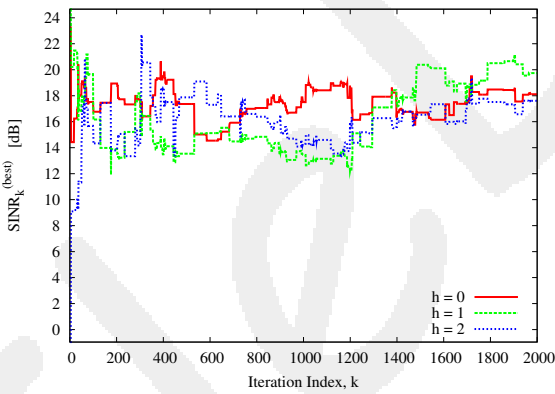


Fig.13 - SINR

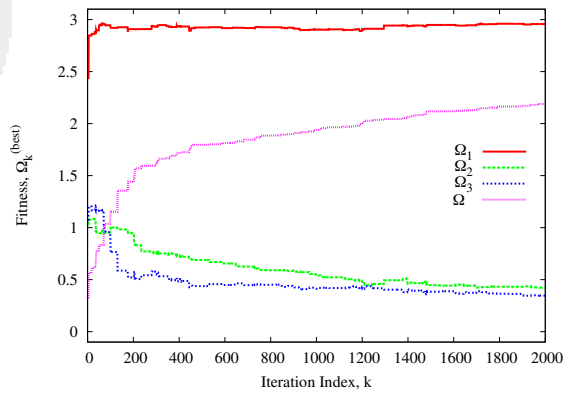


Fig.14 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	18.114464	19.742274	17.611541	-13.293117	-9.263318	-11.797567	-2.606293	-2.739554

Tab. 3 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 2.b

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$

## Results

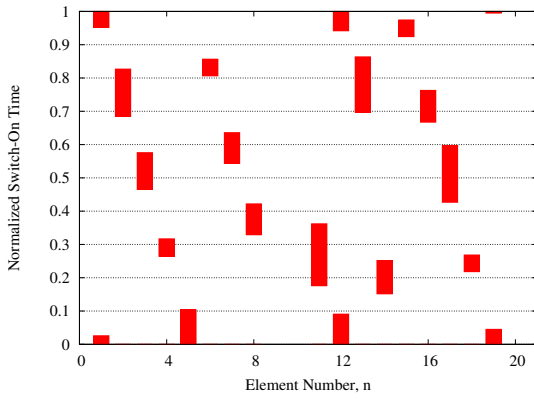


Fig.15 - Pulse Sequence

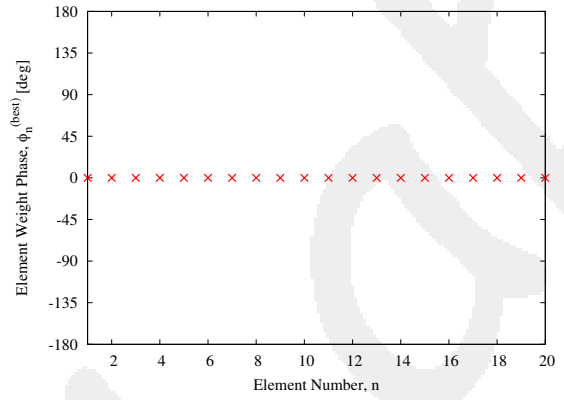


Fig.16 - Phases

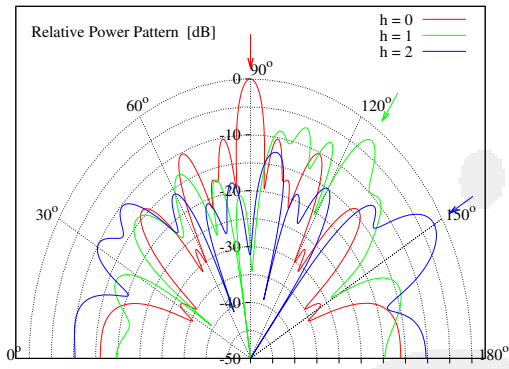


Fig.17 - Pattern

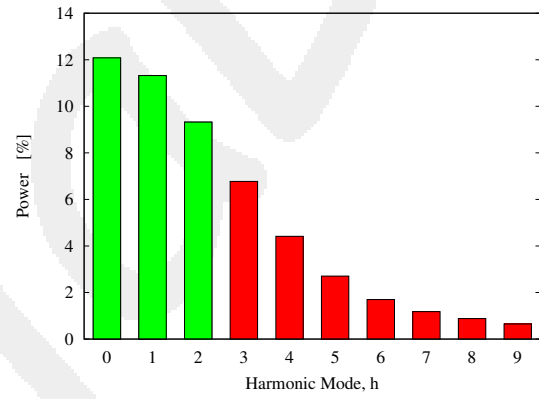


Fig.18 - Sideband Radiation

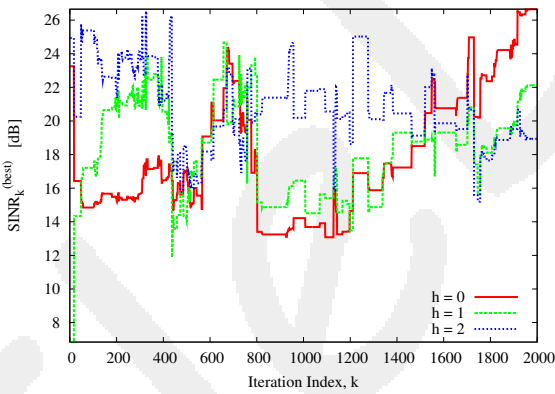


Fig.19 - SINR

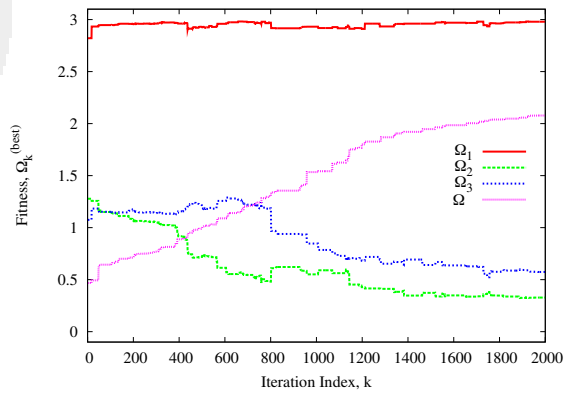


Fig.20 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	26.651382	22.126262	18.937109	-10.289671	-6.873352	-10.045515	-2.410769	-2.085163

Tab. 4 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 2.c

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$
- Considering Non-Uniform Phase relationship

## Results

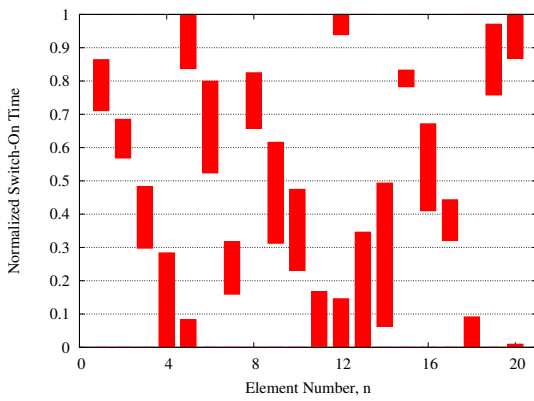


Fig.21 - Pulse Sequence

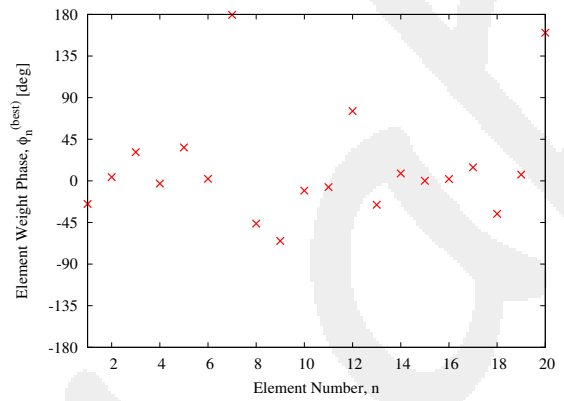


Fig.22 - Phases

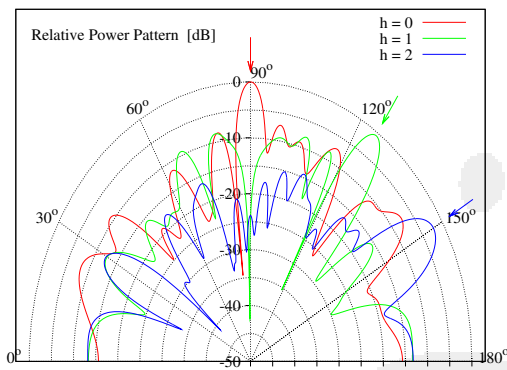


Fig.23 - Pattern

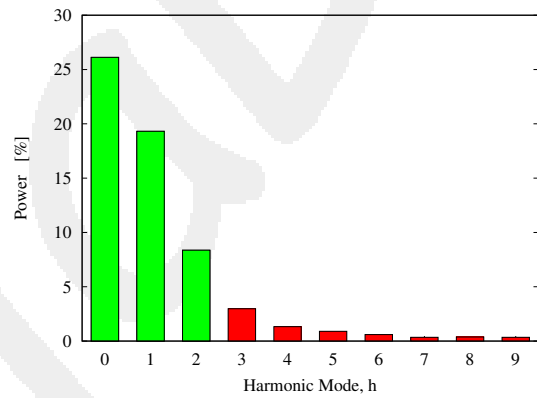


Fig.24 - Sideband Radiation

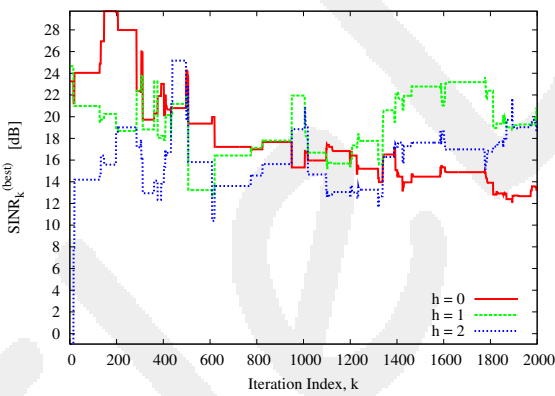


Fig.25 - SINR

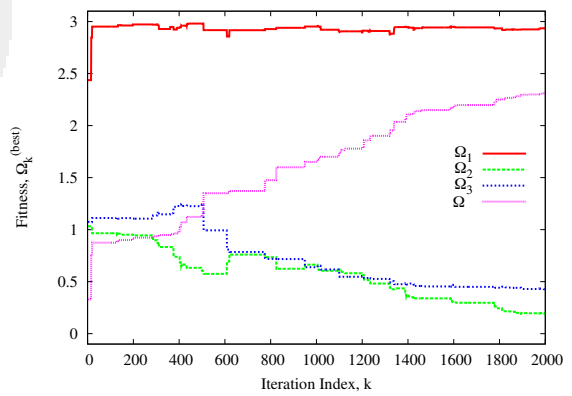


Fig.26 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	13.325438	20.781401	19.066351	-7.220614	-7.979029	-12.584997	-0.650142	-2.298933

Tab. 5 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 2.d

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$
- Non Uniform Phase Considered.

## Results

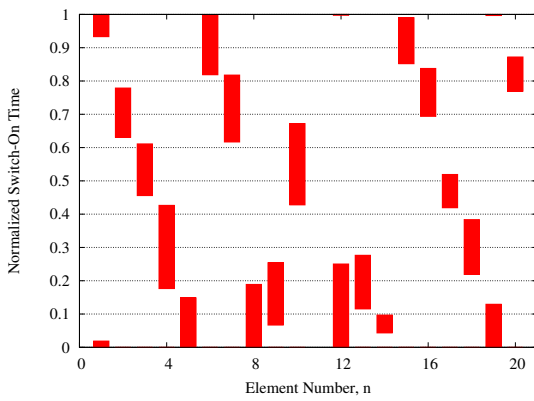


Fig.27 - Pulse Sequence

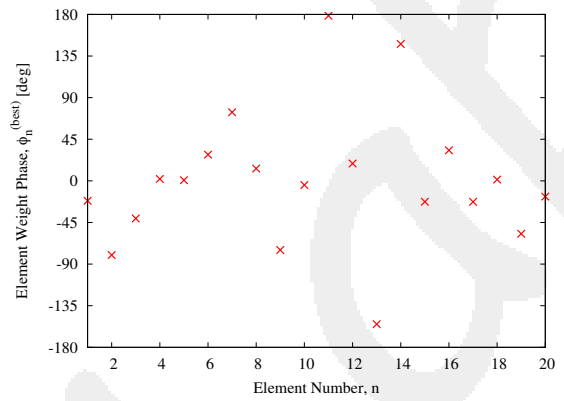


Fig.28 - Phases

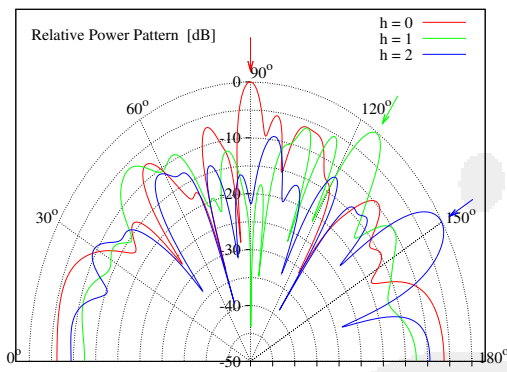


Fig.29 - Pattern

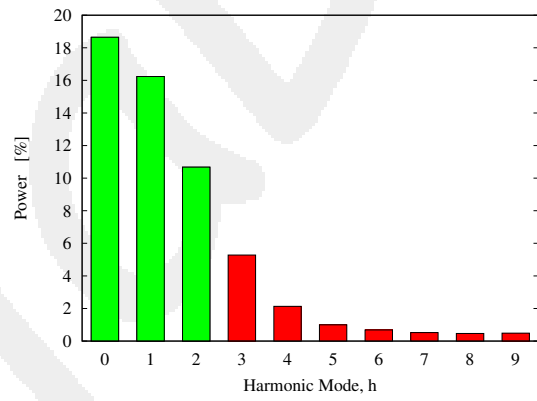


Fig.30 - Sideband Radiation

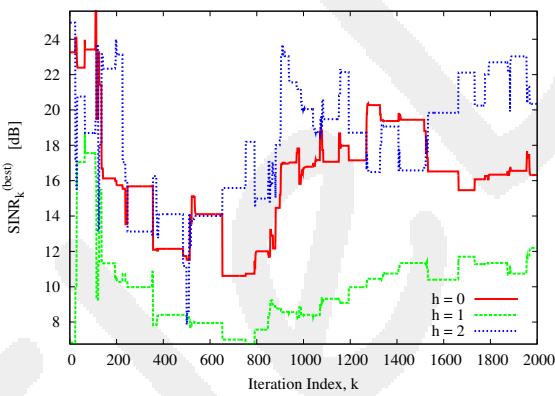


Fig.31 - SINR

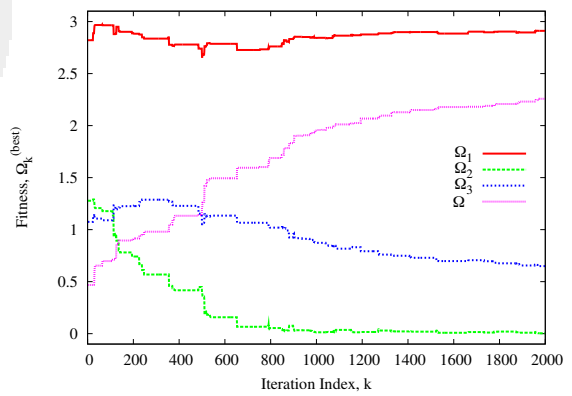


Fig.32 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	16.317202	12.192294	20.348352	-5.643298	-5.503495	-9.359101	-0.239557	-0.161564

Tab. 6 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 3 - $\theta_i = 90^\circ, 65^\circ, 40^\circ$

### TEST CASE 3.a

#### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

#### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 65^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$



## Results

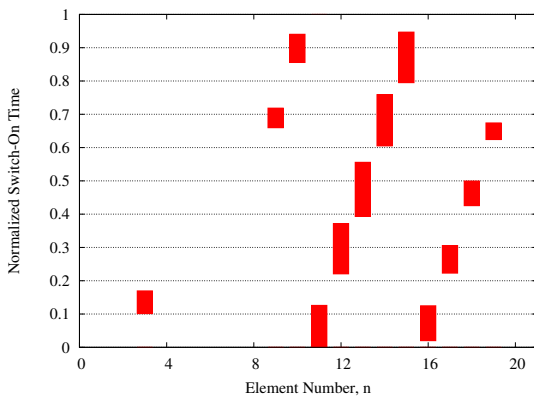


Fig.33 - Pulse Sequence

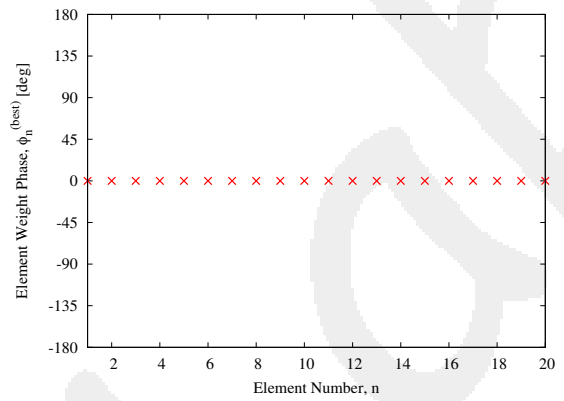


Fig.34 - Phases

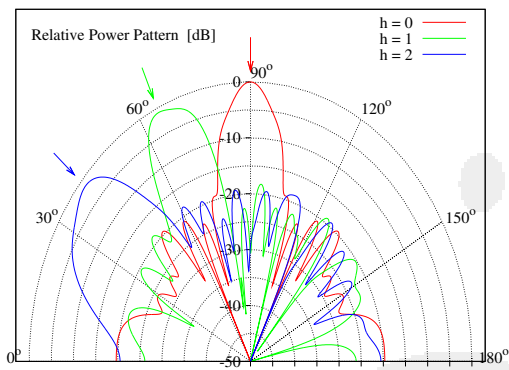


Fig.35 - Pattern

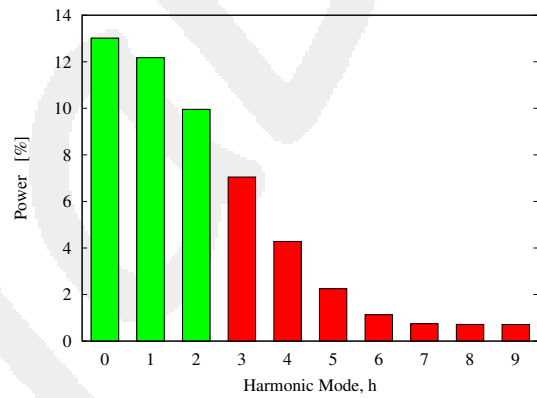


Fig.36 - Sideband Radiation

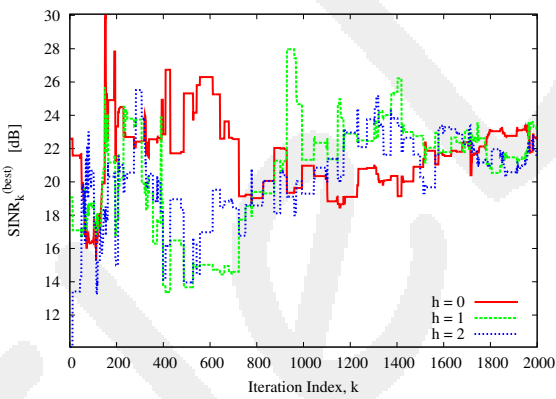


Fig.37 - SINR

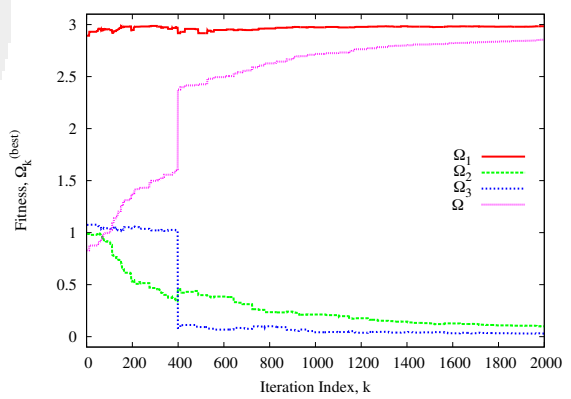


Fig.38 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	22.588477	23.195031	21.542269	-19.495398	-18.221585	-18.477332	-0.880394	-1.284707

Tab. 7 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 3.b

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 65^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$

## Results

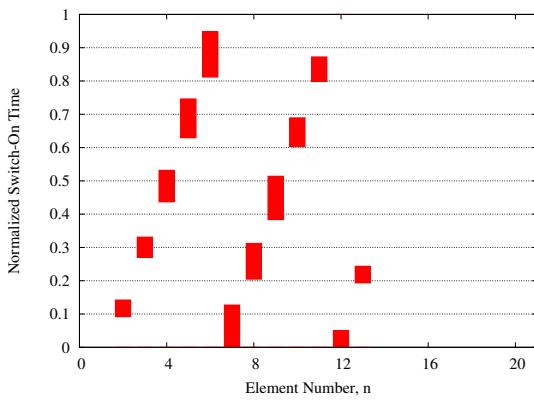


Fig.39 - Pulse Sequence

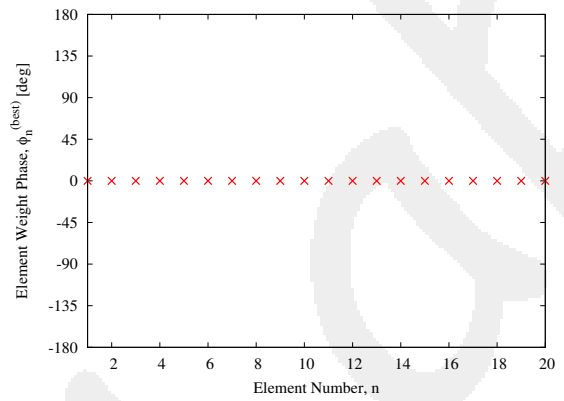


Fig.40 - Phases

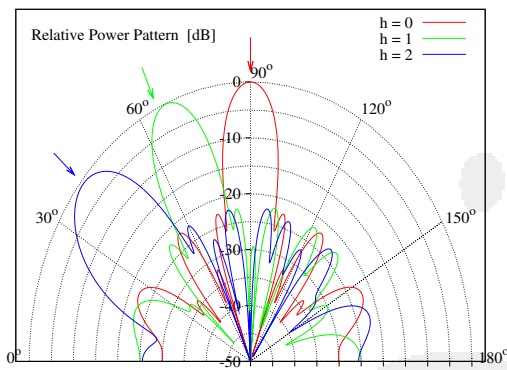


Fig.41 - Pattern

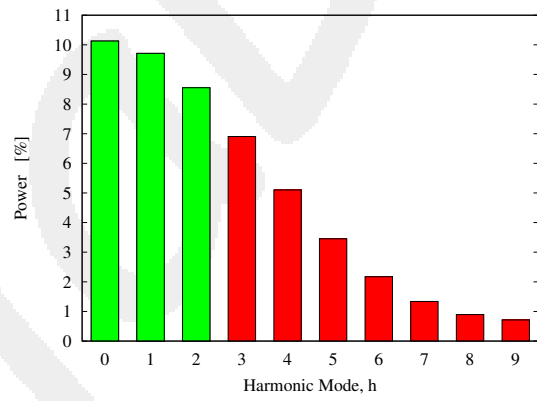


Fig.42 - Sideband Radiation

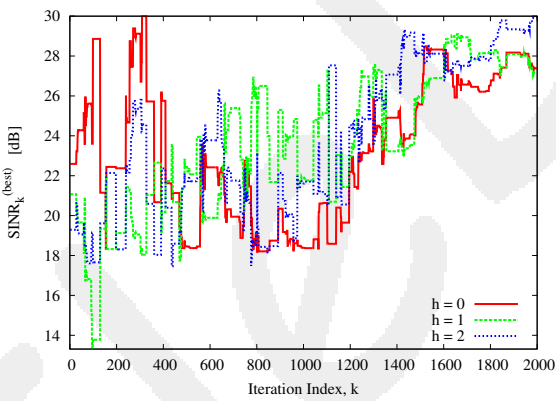


Fig.43 - SINR

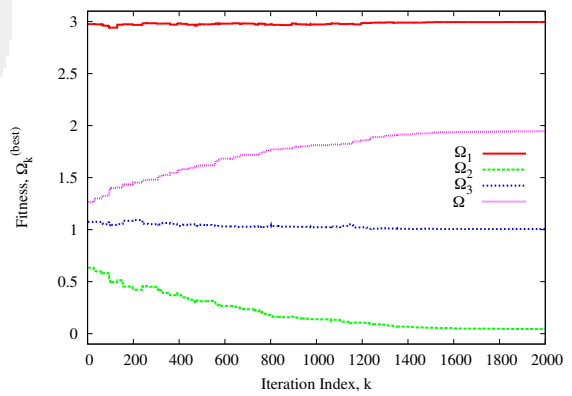


Fig.44 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	27.377289	27.188979	29.932660	-20.091286	-21.052806	-22.057436	-0.719270	-0.653523

Tab. 8 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

### TEST CASE 3.c

#### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

#### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 65^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$
- With a consideration for non uniform phases

## Results

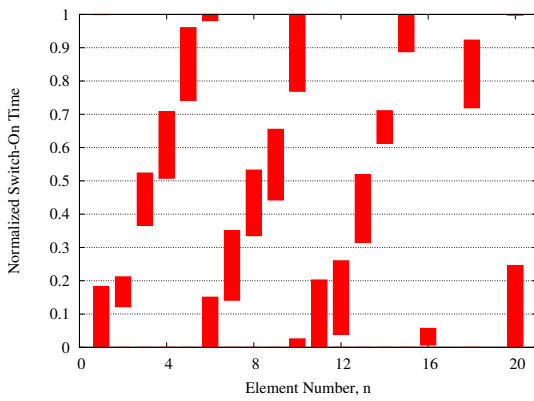


Fig.45 - Pulse Sequence

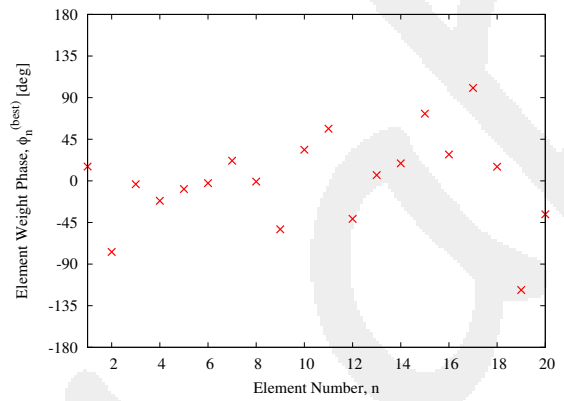


Fig.46 - Phases

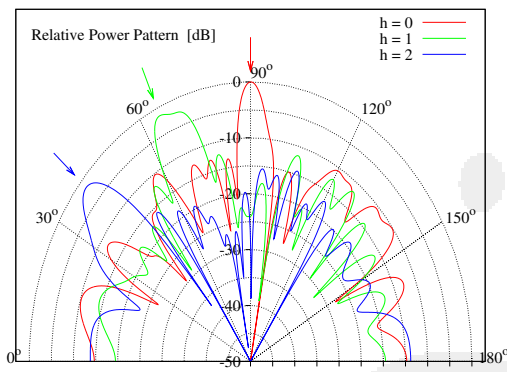


Fig.47 - Pattern

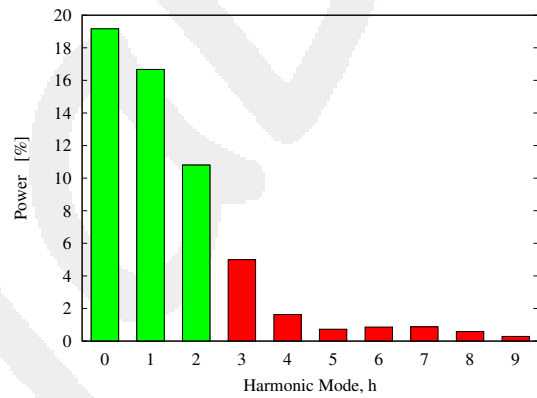


Fig.48 - Sideband Radiation

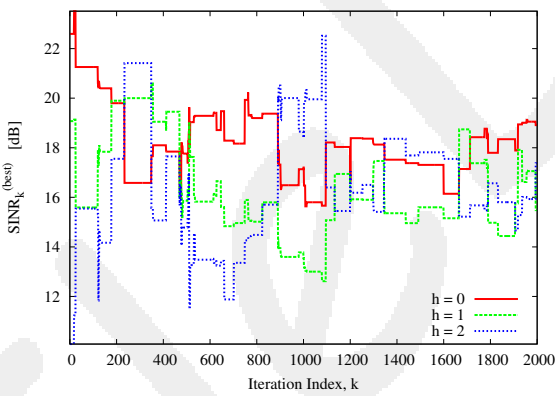


Fig.49 - SINR

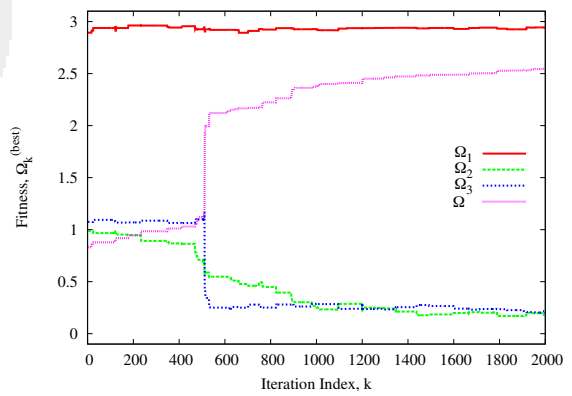


Fig.50 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	18.913158	15.485225	17.373253	-10.192098	-11.667730	-14.366098	-1.512422	-1.583574

Tab. 9 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 3.d

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 65^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -15dB$
- With a consideration for non uniform phases

## Results

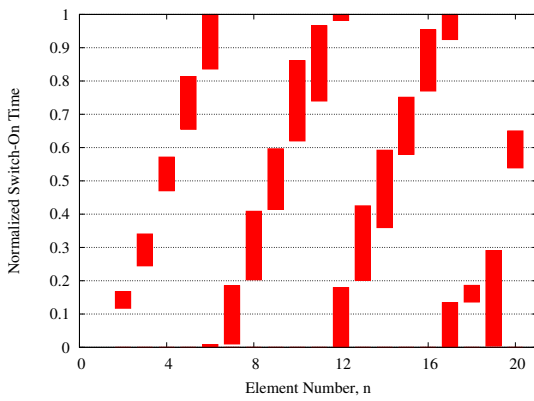


Fig.51 - Pulse Sequence

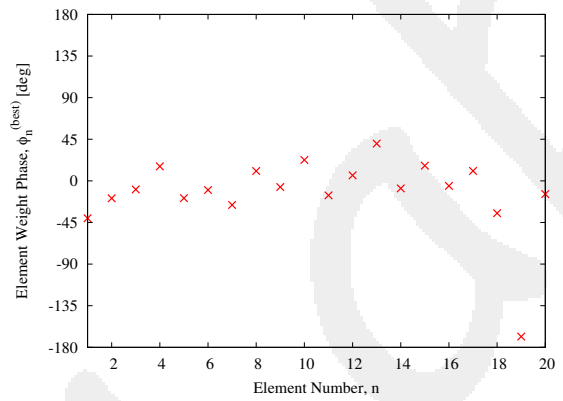


Fig.52 - Phases

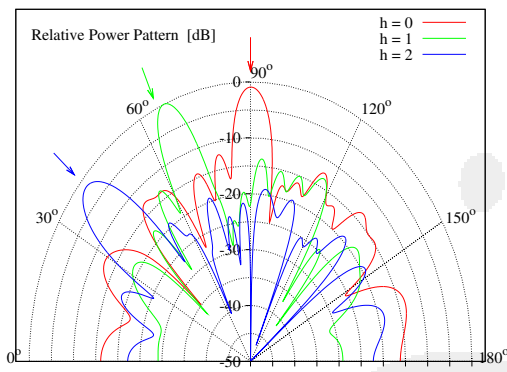


Fig.53 - Pattern

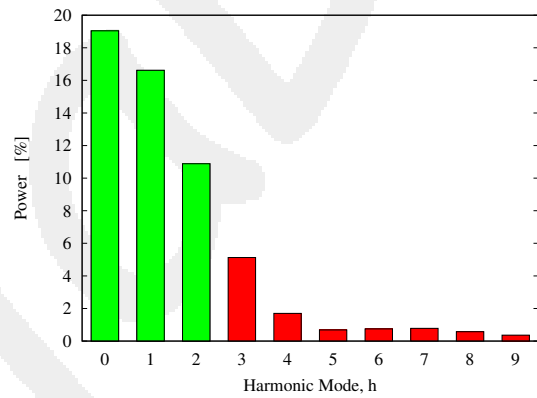


Fig.54 - Sideband Radiation

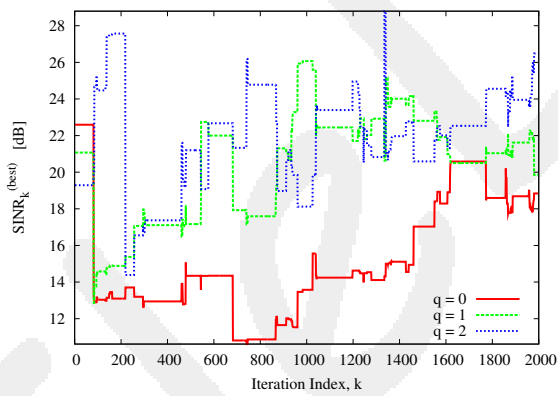


Fig.55 - SINR

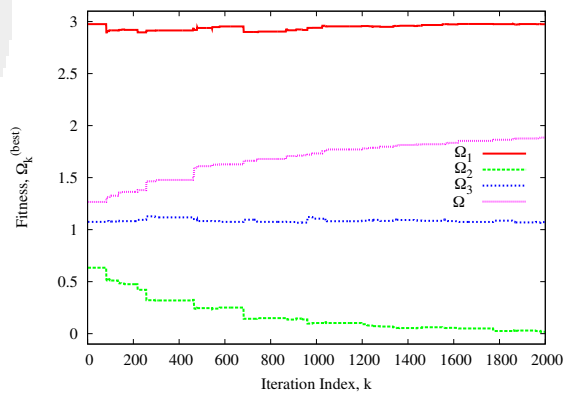


Fig.56 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SBL_{h=1}$	$SBL_{h=2}$
PSO	18.840556	19.852013	26.484680	-12.340148	-10.040477	-10.000552	-0.938150	-1.534277

Tab. 10 - Signal to Noise plus Interferences Ratio ( $SINR$ ), Sidelobe Level ( $SLL$ ) and Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ) and the second harmonic frequency ( $h = 2$ ) (all the values expressed in dB).

## TEST CASE 4 - $\theta_i = 90^\circ, 125^\circ, 150^\circ, 40^\circ$

### TEST CASE 4.a

#### Goal

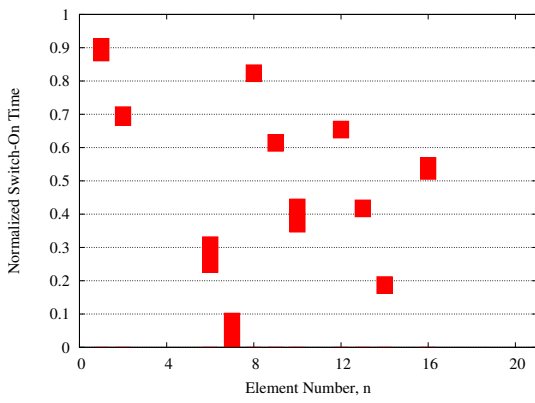
This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

#### Test Case Description

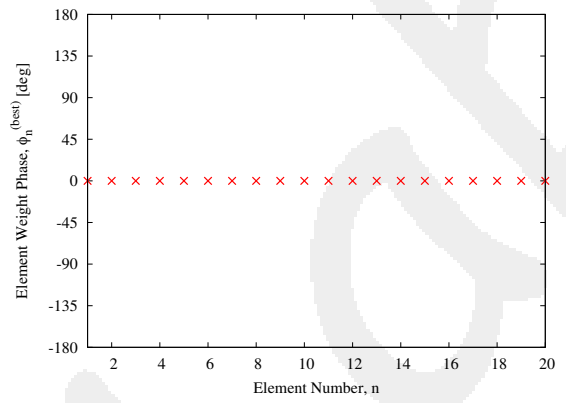
- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$



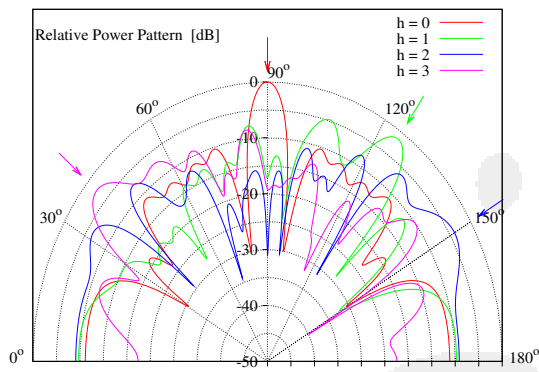
# Results



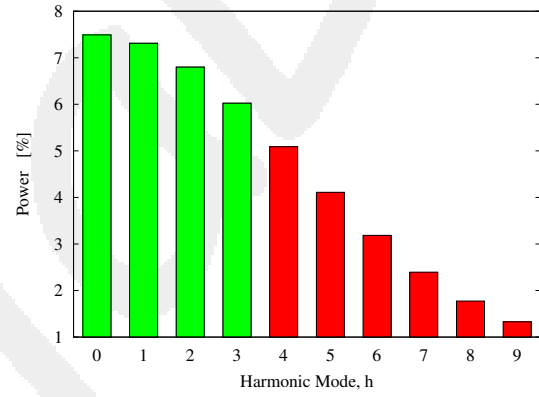
**Fig.57 - Pulse Sequence**



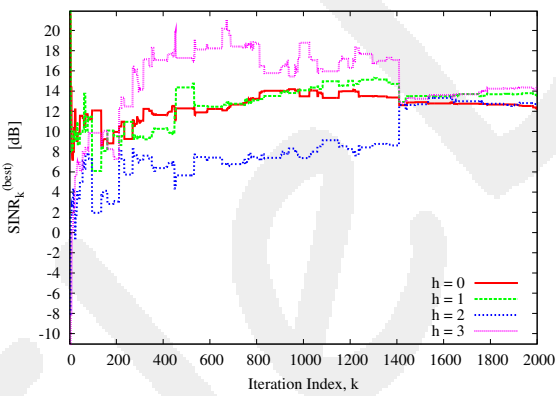
**Fig.58 - Phases**



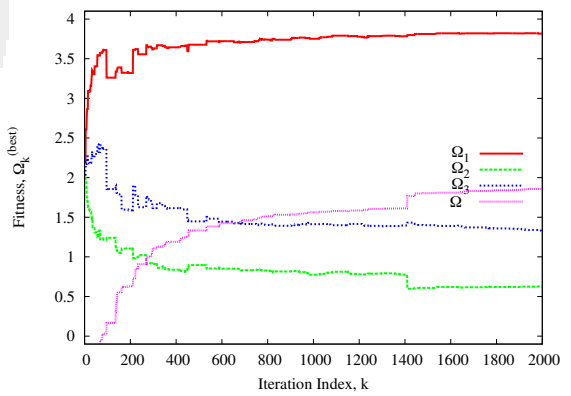
**Fig.59 - Pattern**



**Fig.60 - Sideband Radiation**



**Fig.61 - SINR**



**Fig.62 - Fitness**

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	12.200476	13.522266	12.794237	14.502454

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-10.371668	-4.788086	-6.126273	-7.196168

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-1.455780	-3.981959	-2.469827

Tab. 11 - (a) Signal to Noise plus Interferences Ratio ( $SINR$ ), (b) Sidelobe Level ( $SLL$ ) and (c) Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).

## TEST CASE 4.b

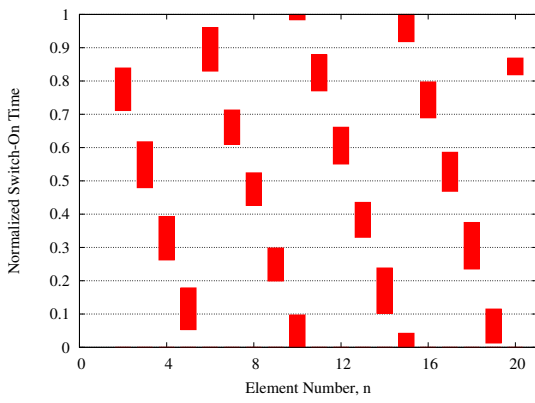
### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

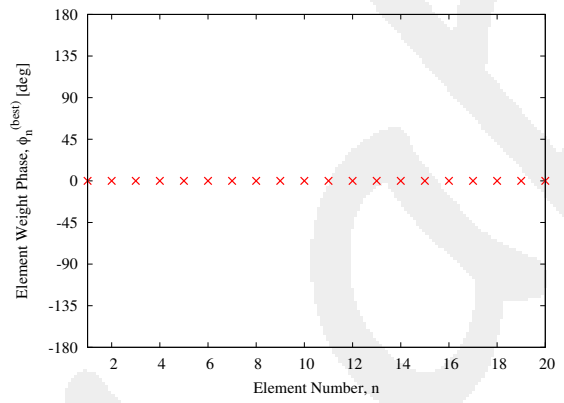
### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$

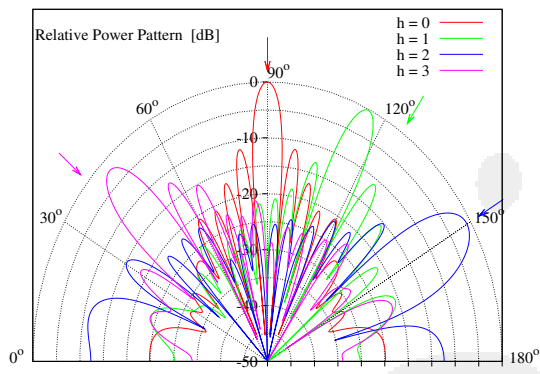
## Results



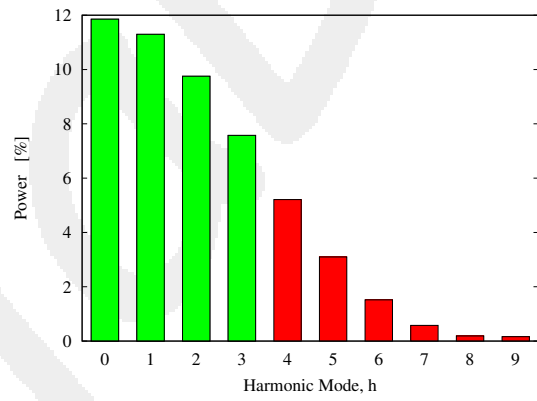
**Fig.63 - Pulse Sequence**



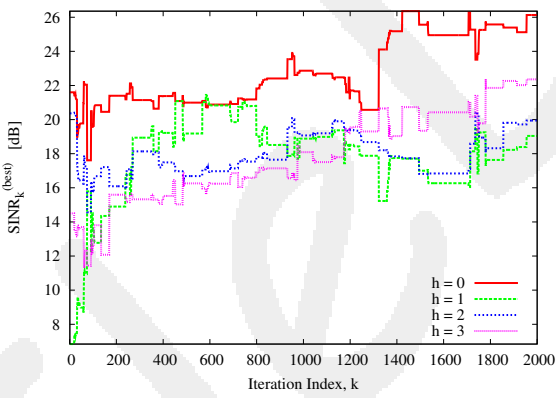
**Fig.64 - Phases**



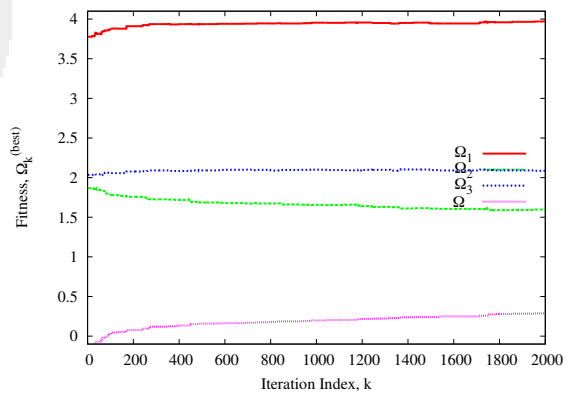
**Fig.65 - Pattern**



**Fig.66 - Sideband Radiation**



**Fig.67 - SINR**



**Fig.68 - Fitness**

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	26.131057	19.043202	19.944810	22.360690

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-11.645709	-0.223703	-11.120879	-11.871134

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-11.521016	-0.910255	-7.734555

**Tab. 12 - (a) Signal to Noise plus Interferences Ratio ( $SINR$ ), (b) Sidelobe Level ( $SLL$ ) and (c) Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).**

## TEST CASE 4.c

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{3}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$

## Results

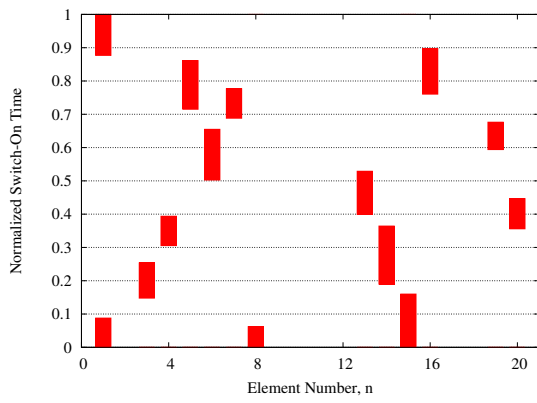


Fig.69 - Pulse Sequence

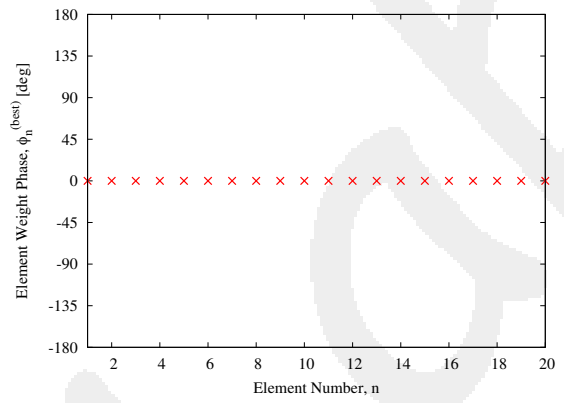


Fig.70 - Phases

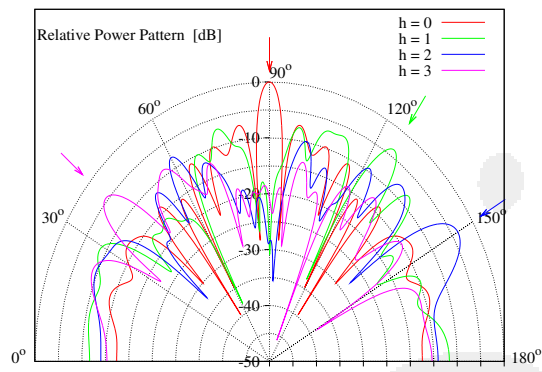


Fig.71 - Pattern

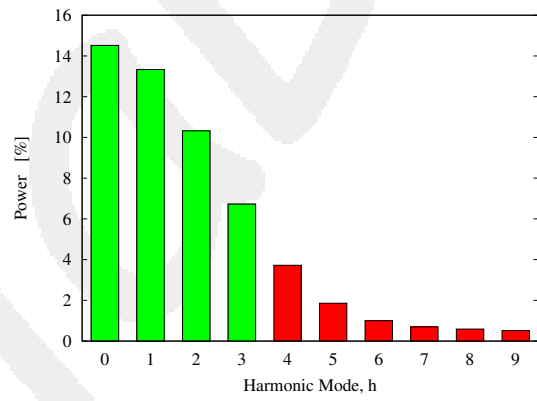


Fig.72 - Sideband Radiation

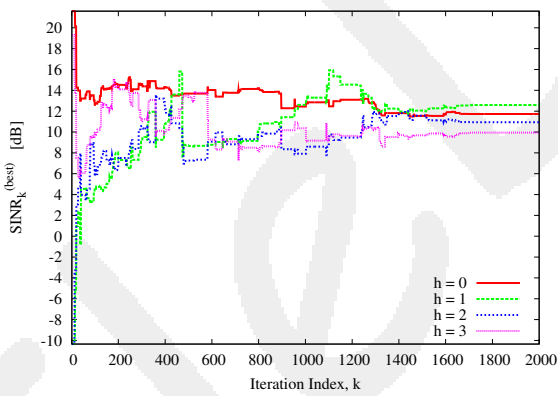


Fig.73 - SINR

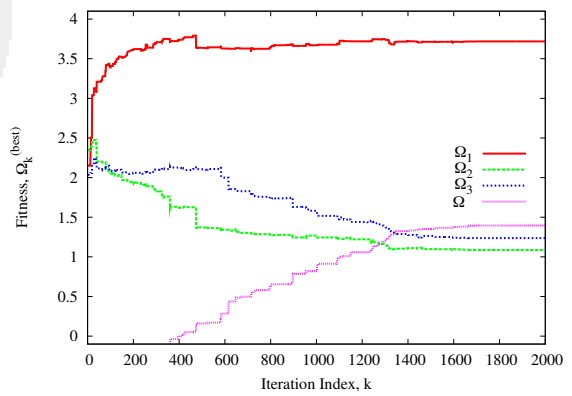


Fig.74 - Fitness

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	11.728310	12.585958	10.940509	9.942525

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-7.281876	-6.764363	-7.879346	-8.951906

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-3.801415	-3.683281	-4.524662

**Tab. 13 - (a) Signal to Noise plus Interferences Ratio ( $SINR$ ), (b) Sidelobe Level ( $SLL$ ) and (c) Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).**



## TEST CASE 4.d

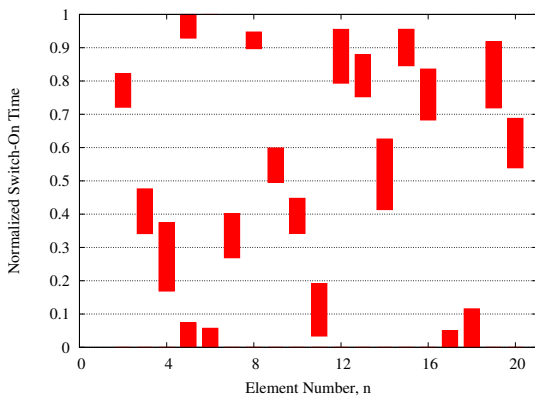
### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

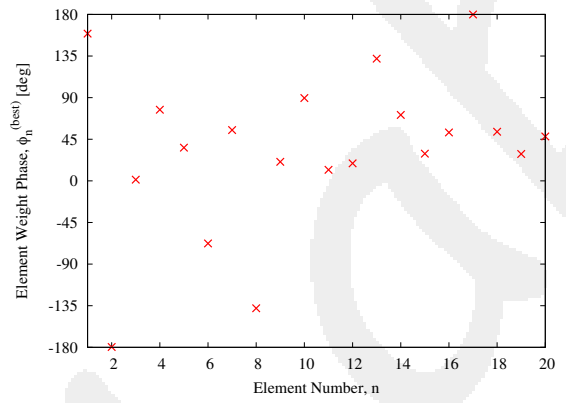
### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{1}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$
- With a consideration for non uniform phases

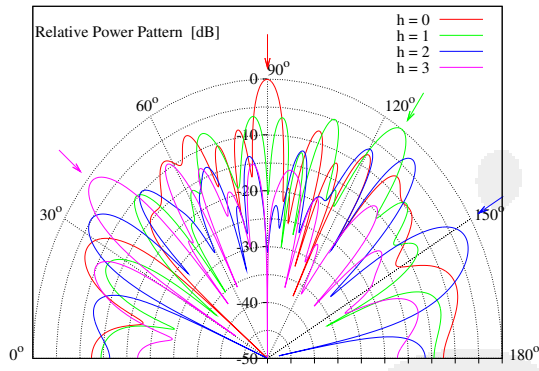
# Results



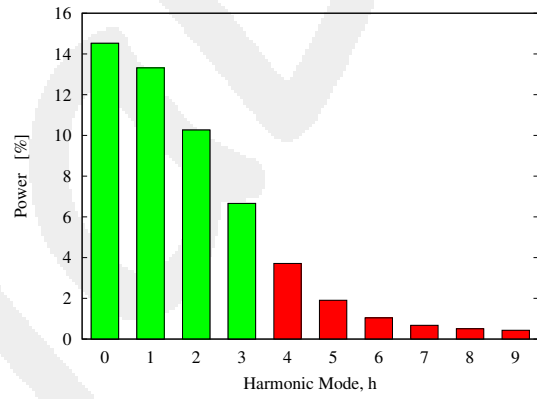
**Fig.75 - Pulse Sequence**



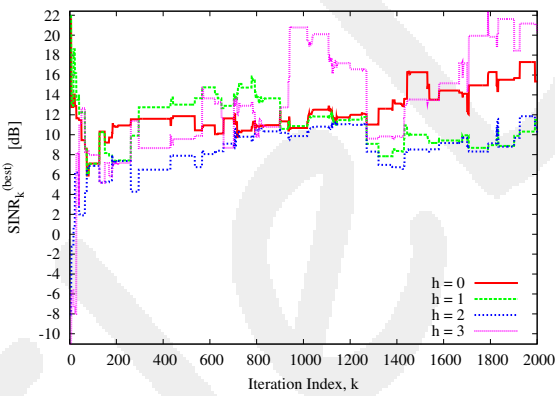
**Fig.76 - Phases**



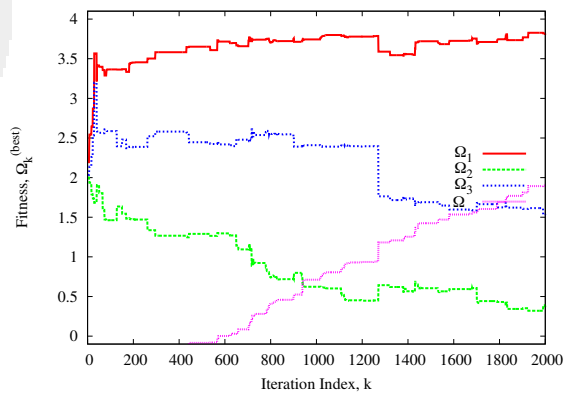
**Fig.77 - Pattern**



**Fig.78 - Sideband Radiation**



**Fig.79 - SINR**



**Fig.80 - Fitness**

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	15.397695	11.130589	10.028021	20.257037

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-6.579566	-4.910518	-2.900814	-9.257707

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-0.002083	-3.568271	-0.932598

Tab. 14 - (a) Signal to Noise plus Interferences Ratio (*SINR*), (b) Sidelobe Level (*SLL*) and (c) Sideband Level (*SBL*) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).

## TEST CASE 4.e

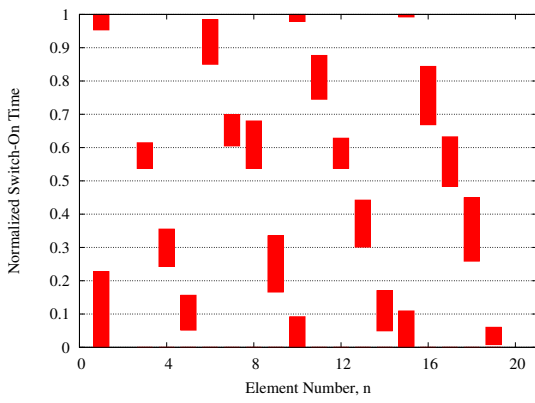
### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

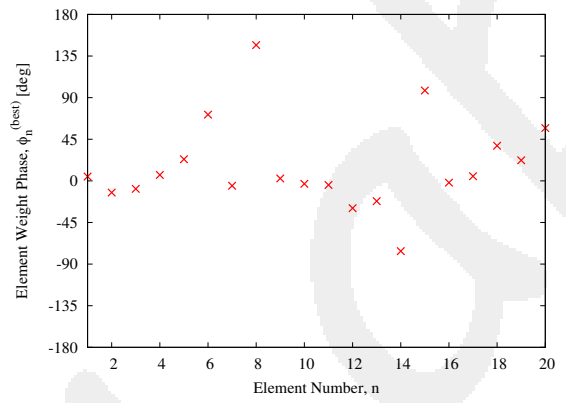
### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$ ,  $\eta_3 = 1$
- $\mathbf{h} = \mathbf{2}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$
- With a consideration for non uniform phases

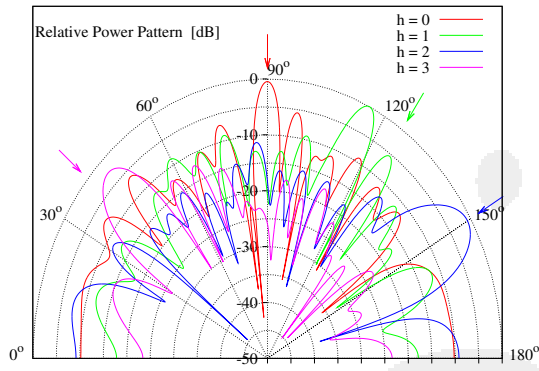
# Results



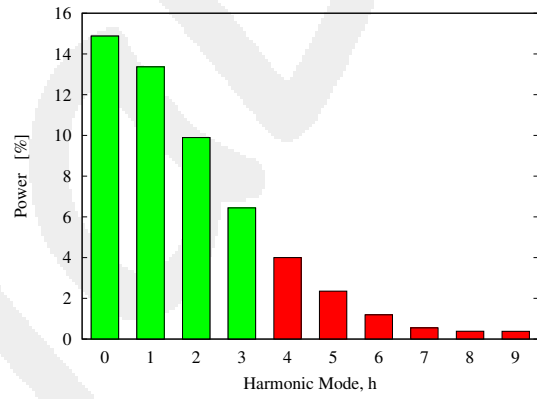
**Fig.81 - Pulse Sequence**



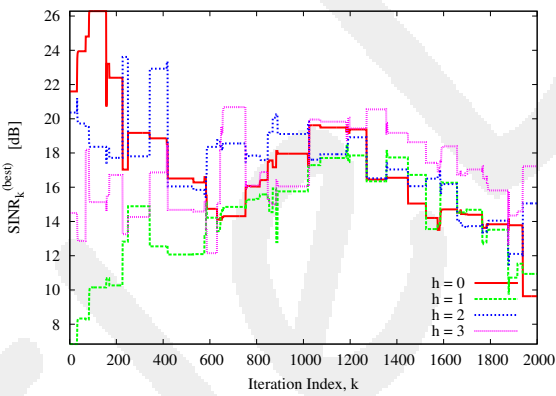
**Fig.82 - Phases**



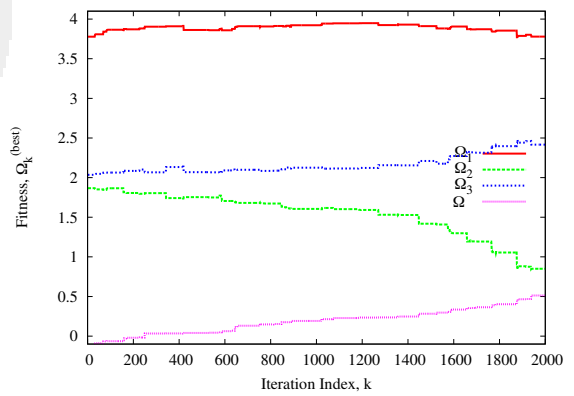
**Fig.83 - Pattern**



**Fig.84 - Sideband Radiation**



**Fig.85 - SINR**



**Fig.86 - Fitness**

	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	9.634997	10.939047	15.059042	17.227547

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-5.610479	-0.000000	-7.505694	-12.161662

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-5.721086	-0.801938	-4.314914

**Tab. 15 - (a) Signal to Noise plus Interferences Ratio ( $SINR$ ), (b) Sidelobe Level ( $SLL$ ) and (c) Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).**

## TEST CASE 4.f

### Goal

This test case is used as a reference to observe the behaviour of the synthesis with regards to using different PSO parameters.

### Test Case Description

- Signal  $i = 0$ , desired in the fundamental frequency,  $h = 0$ , with DOA:  $\theta = 90^\circ$
- Signal  $i = 1$ , desired in the harmonic frequency,  $h = 1$  with DOA:  $\theta = 125^\circ$
- Signal  $i = 2$ , desired in the harmonic frequency,  $h = 2$  with DOA:  $\theta = 150^\circ$
- Signal  $i = 3$ , desired in the harmonic frequency,  $h = 3$  with DOA:  $\theta = 40^\circ$
- PSO iterations: 2000
- Cost function weights:  $\eta_1 = 1$ ,  $\eta_2 = 1$   $\eta_3 = 1$
- $\mathbf{h} = \mathbf{3}$  is the harmonic index in which signal  $i$  will be received, and  $\theta^i$  is the DOA of this signal. The initial pattern in the  $h^{th}$  harmonic will be directed to this angle.
- Reference Side lobe level:  $SLL_{ref} = -10dB$
- With a consideration for non uniform phases

## Results

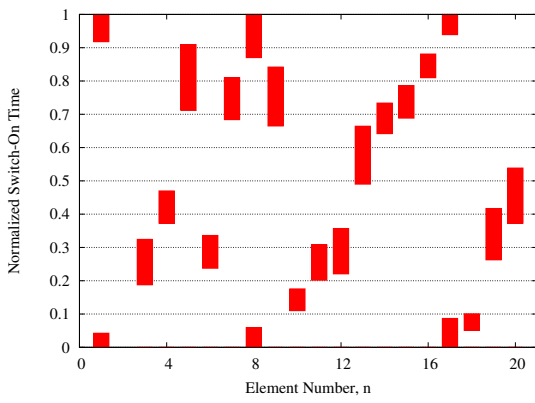


Fig.87 - Pulse Sequence

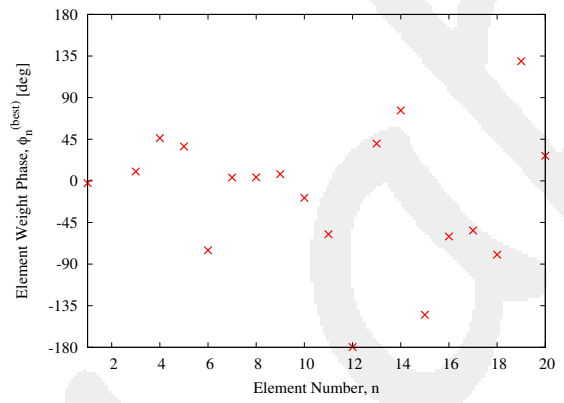


Fig.88 - Phases

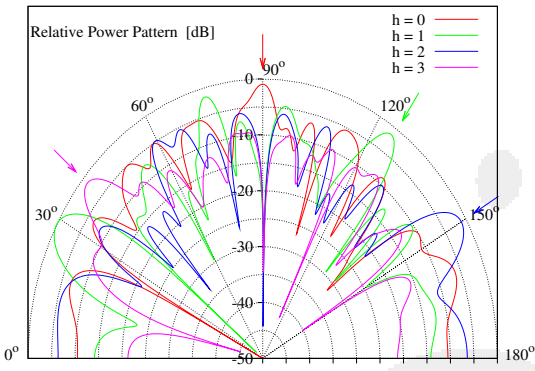


Fig.89 - Pattern

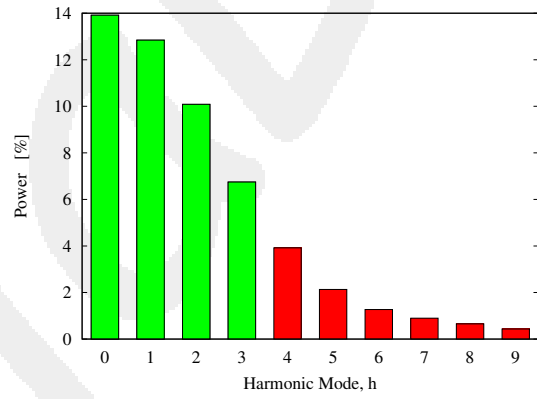


Fig.90 - Sideband Radiation

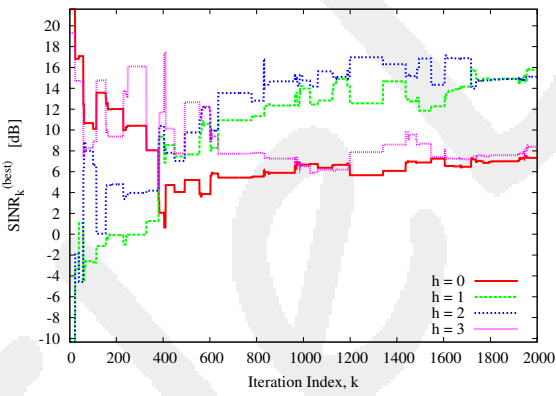


Fig.91 - SINR

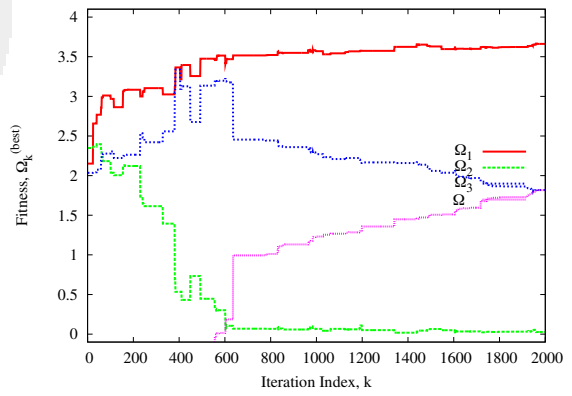


Fig.92 - Fitness



	$SINR_{h=0}$	$SINR_{h=1}$	$SINR_{h=2}$	$SINR_{h=3}$
<i>PSO</i>	7.332266	15.787774	15.107539	8.394382

	$SLL_{h=0}$	$SLL_{h=1}$	$SLL_{h=2}$	$SLL_{h=3}$
<i>PSO</i>	-5.057743	0.000000	-4.884350	-9.272404

	$SBL_{h=1}$	$SBL_{h=2}$	$SBL_{h=3}$
<i>PSO</i>	-1.657266	-1.137674	-1.225073

Tab. 16 - (a) Signal to Noise plus Interferences Ratio ( $SINR$ ), (b) Sidelobe Level ( $SLL$ ) and (c) Sideband Level ( $SBL$ ) related to the central frequency ( $h = 0$ ), the first harmonic frequency ( $h = 1$ ), the second harmonic frequency ( $h = 2$ ) and the third harmonic frequency ( $h = 3$ ) (all the values expressed in dB).

## References

- [1] P. Rocca, G. Oliveri, and A. Massa, "Differential Evolution as applied to electromagnetics," *IEEE Antennas Propag. Mag.*, vol. 53, no. 1, pp. 38-49, Feb. 2011.
- [2] E. T. Bekele, L. Poli, M. D'Urso, P. Rocca, and A. Massa, "Pulse-shaping strategy for time modulated arrays - Analysis and design," *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3525-3537, July 2013.
- [3] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Harmonic beamforming in time-modulated linear arrays," *IEEE Trans. Antennas Propag.*, vol. 59, no. 7, pp. 2538-2545, Jul. 2011.
- [4] L. Poli, P. Rocca, L. Manica, and A. Massa, "Handling sideband radiations in time-modulated arrays through particle swarm optimization," *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1408-1411, Apr. 2010.
- [5] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "Adaptive nulling in time-varying scenarios through time-modulated linear arrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 101-104, 2012.
- [6] P. Rocca, L. Poli, L. Manica, and A. Massa, "Synthesis of monopulse time-modulated planar arrays with controlled sideband radiation," *IET Radar, Sonar & Navigation*, vol. 6, no. 6, pp. 432-442, 2012.
- [7] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Adaptive nulling in time-modulated linear arrays with minimum power losses," *IET Microwaves, Antennas & Propagation*, vol. 5, no. 2, pp. 157-166, 2011.
- [8] L. Poli, P. Rocca, L. Manica, and A. Massa, "Time modulated planar arrays - Analysis and optimization of the sideband radiations," *IET Microwaves, Antennas & Propagation*, vol. 4, no. 9, pp. 1165-1171, 2010.
- [9] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "Synthesis of time-modulated planar arrays with controlled harmonic radiations," *Journal of Electromagnetic Waves and Applications*, vol. 24, no. 5/6, pp. 827-838, 2010.
- [10] L. Manica, P. Rocca, L. Poli, and A. Massa, "Almost time-independent performance in time-modulated linear arrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 8, pp. 843-846, 2009.
- [11] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Failure correction in time-modulated linear arrays," *IET Radar, Sonar & Navigation*, vol. 8, no. 3, pp. 195-201, Mar. 2014.
- [12] P. Rocca, Q. Zhu, E. T. Bekele, S. Yang, and A. Massa, "4D arrays as enabling technology for cognitive radio systems," *IEEE Transactions on Antennas and Propagation - Special Issue on "Antenna Systems and Propagation for Cognitive Radio"*, vol. 62, no. 3, pp. 1102-1116, Mar. 2014.