

4D Arrays for New Generation MIMO Applications

E. T. Bekele, P. Rocca, A. Massa

Abstract

In this report, an innovative application of 4-D arrays is proposed and assessed. The possibility to simultaneously receive multiple signals impinging on the antenna from different directions such to increase the wireless system throughput by means of a proper definition of the pulse sequence controlling the on-off behavior of the RF switches is investigated.

Introduction

The cost function to be used in the simulations is defined as follows:

$$\begin{aligned}
 \Omega &= \eta_1 \Omega_1 - \eta_2 \Omega_2 - \eta_3 \Omega_3 \\
 \Omega_1 &= \sum_{h=-H}^H \left[\frac{P_d^{(h)}}{P_d^{(h)} + P_u^{(h)} + P_n^{(h)}} \delta(h) \right] \\
 \Omega_2 &= \sum_{h=-H}^H \left[\left(\frac{P_d^{max} - P_d^{(h)}}{P_d^{max}} \right)^2 \delta(h) \right] \\
 \Omega_3 &= \sum_{h=-H}^H \left[\left(\frac{SLL^{(h)} - SLL_{ref}^{(h)}}{SLL_{peak}^{(h)} - SLL_{ref}^{(h)}} \right)^2 \Upsilon \left(SLL^{(h)} - SLL_{ref}^{(h)} \right) \delta(h) \right]
 \end{aligned}$$

Cost Function

where:

- $P_d^{(h)}$ is the power received in the h^{th} harmonic, from the signal desired in this harmonic.
- $P_u^{(h)} = \sum_{i=0}^{I-1} P_i^{(h)}$ is the sum of power received in the h^{th} harmonic from all signal sources but the desired signal in this harmonic.
 $i \neq d$
- $P_n^{(h)} = \frac{1}{2} \underline{W}_{(h)}^H \Phi_n \underline{W}_{(h)}$ is the noise power captured in the h^{th} harmonic.
- $\underline{W}_{(h)}$ is a column vector of complex harmonic element weights, whose n^{th} element $W_{(h)}[n]$ is given as:
 - $W_{(h)}[n] = A_n U_{hn} e^{j h \omega_p t}$
 - A_n is complex static element weight.
 - U_{hn} is the complex fourier coefficient of the time modulating function $u_n(t)$.
- $\underline{W}_{(h)}^H$ is the hermetian transpose of $\underline{W}_{(h)}$.
- Φ_n is the noise covariance matrix.
- $P_i^{(h)}$ is the power received from signal source i , in the h^{th} harmonic.
- P_d^{max} is the maximum of all the desired signals $P_d^{(h)}$.
- $\delta(h) = \begin{cases} 1 & \text{if } h \text{ is included in the synthesis} \\ 0 & \text{otherwise} \end{cases}$
- Υ is the Heaviside function
- $\eta_1 \in [0, 1]$, $\eta_2 \in [0, 1]$ $\eta_3 \in [0, 1]$ are the weights of the components of the cost function.
- $SLL^{(h)}$ is the side lobe level of the h^{th} harmonic beam pattern.
- $SLL_{peak}^{(h)}$ is the peak of the pattern of the h^{th} harmonic beam.
- $SLL_{ref}^{(h)}$ is a reference level in the h^{th} harmonic beam pattern.

Simulation Parameters

The following parameters are common to all simulations.

- Isotropic Array Elements: $N = 20$
- Uniformly distributed along the z axis: $x_n = 0$, $y_n = 0$, $z_n = \frac{n\lambda}{2}$

- Uniform amplitude weighting of elements: $\alpha_n = 1$
- Reference Side lobe level: $SLL_{ref} = -15dB$
- Cost function weights: $\eta_1 = 1, \eta_2 = 1, \eta_3 = 1$
- PSO Parameters
 - Number of Variables: $X = 40$ ($\tau_n, i_n^r, n = 1, 2, \dots, N$)
 - Swarm Size: 40
 - Seed of Random Generator: 2500
 - PSO iterations: 2000
 - $w = 0.4$
 - $c_1 = c_2 = 2$
- Signal and Noise parameters
 - Two Signals
 - Harmonic Index: $h = 0, 1$
 - Amplitude and phase for all Signal Sources: $S_i = 1$
 - Noise Power: $\varphi_n = -20dB$
 - Noise Covariance Matrix: $\Phi_n = \varphi_n 1^N$

TEST CASE 1 - Two Signals

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 90^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta = 61.3^\circ$
- Uniform phase weighting of elements: $\varphi_n = 0$
- Initial Pulse width¹: $\tau_n = 0.5$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 1$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

¹ $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = 0.5$ maximizes the array factor, thus the power of the first harmonic, $|h| = 1$.

Results

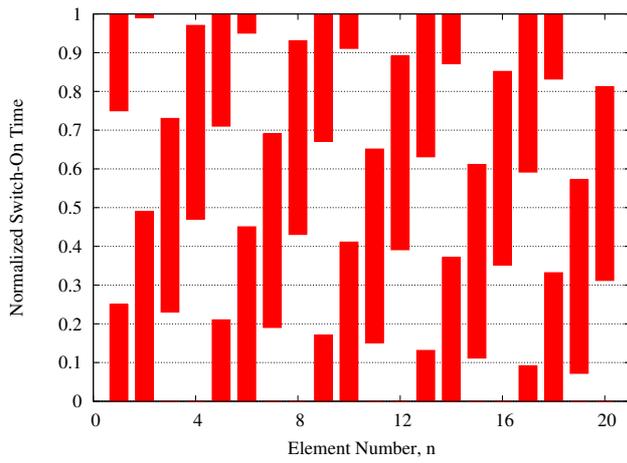


Fig.1 - Initial Pulse Sequence

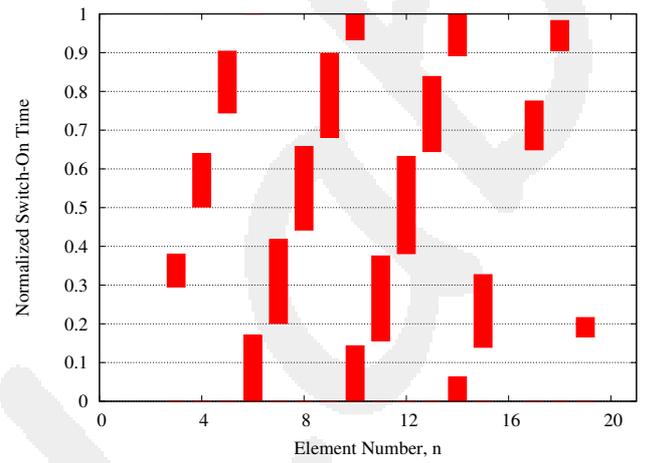


Fig.2 - Optimized Pulse Sequence

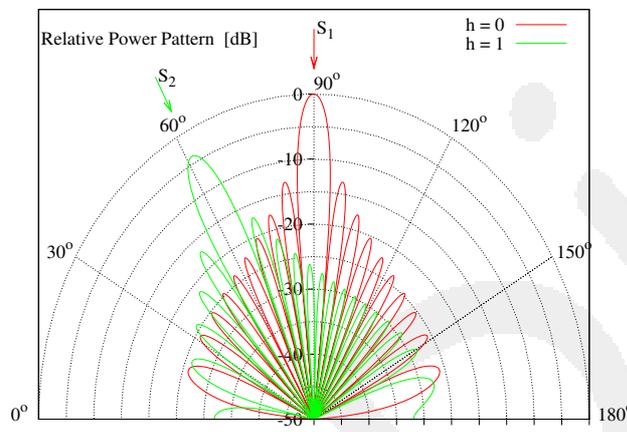


Fig.3 - Initial Pattern

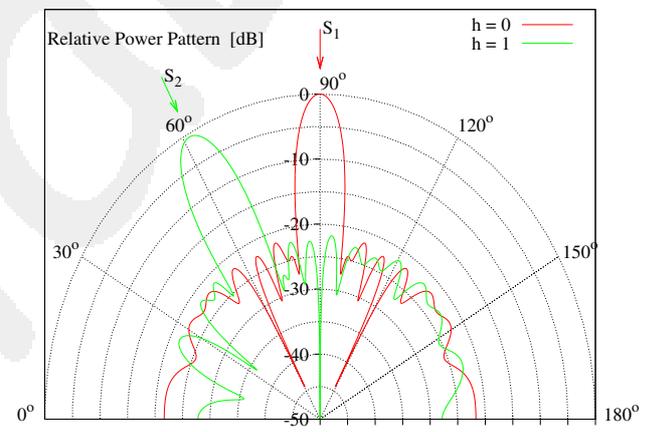


Fig.3 - Optimized Pattern

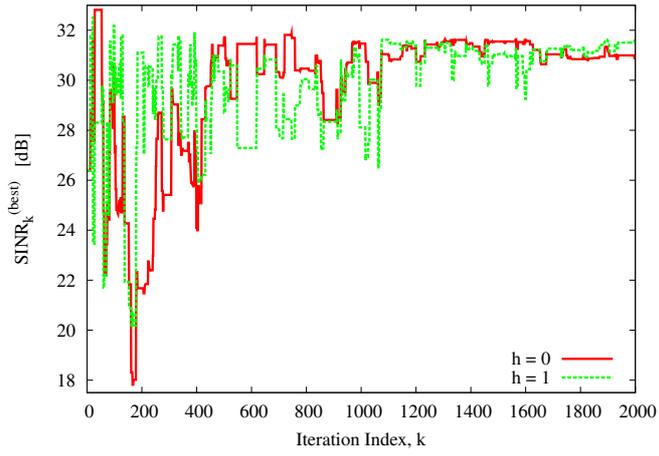


Fig.5 - SINR

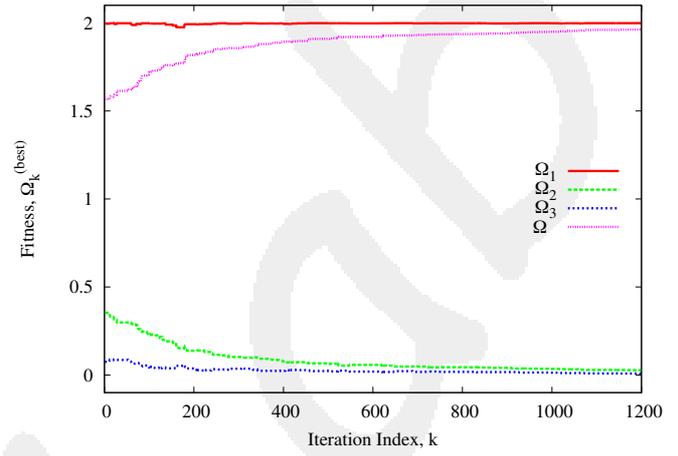


Fig.6 - Fitness

TEST CASE 2 - Two Signals - Time Varying Scenario

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 90^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta \in [5^\circ, 85^\circ] \cup [95^\circ, 175^\circ]$
- Uniform phase weighting of elements: $\varphi_n = 0$
- Initial Pulse width²: $\tau_n = 0.5$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 1$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

² $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = 0.5$ maximizes the array factor, thus the power of the first harmonic, $|h| = 1$.

Results

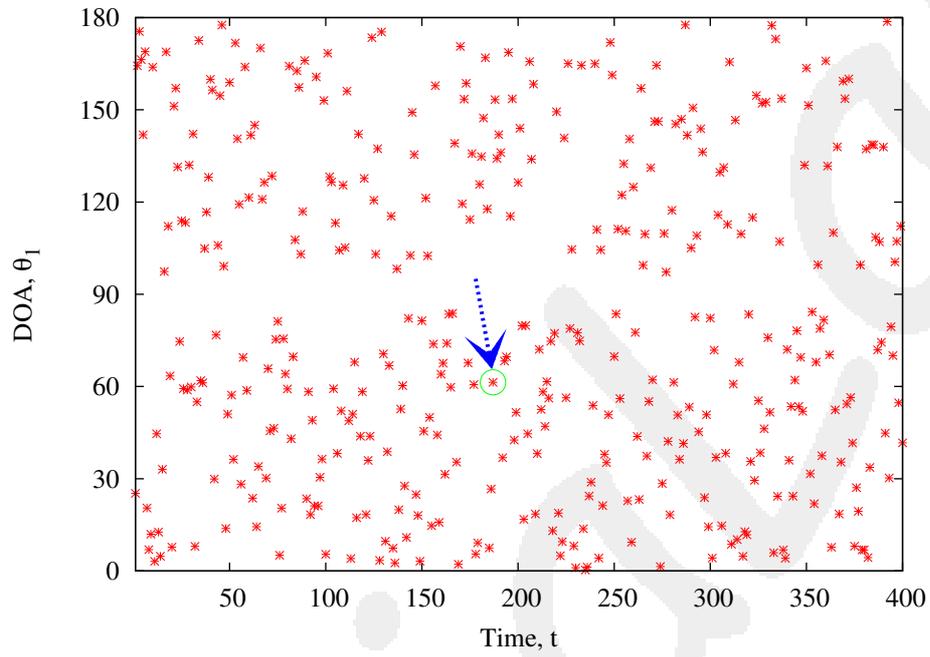


Fig.7 - Time Varying DOA

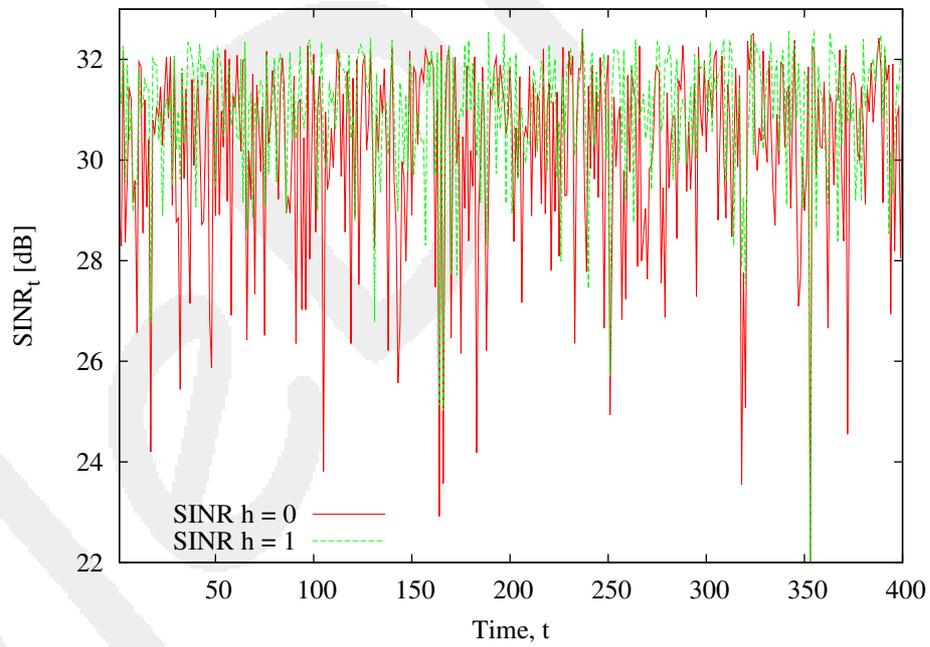


Fig.8 - Time Varying SINR

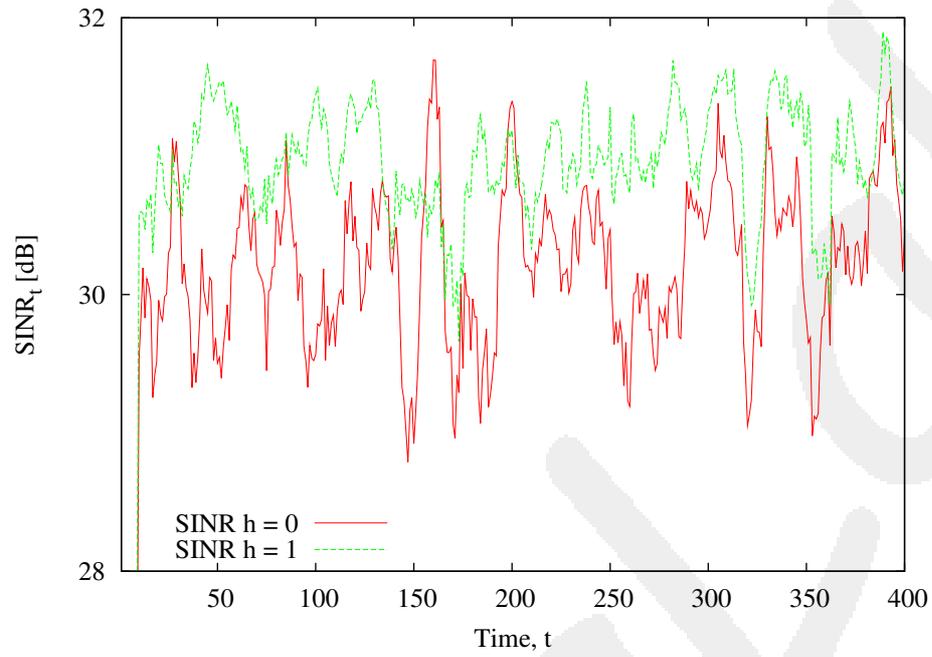


Fig.9 - Filtered Time Varying SINR

	<i>SINR</i> [dB]			
	Average	Std.	Min	Max
Signal $i = 1$	30.19	1.90	21.93	32.60
Signal $i = 2$	30.97	1.29	21.6	32.57

TEST CASE 2.a - Two Signals and $\theta_{(h=0)} \neq 90^\circ$

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 47^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta = 120^\circ$
- Initial Pulse width³: $\tau_n = 0.5$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 1$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

³ $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = 0.5$ maximizes the array factor, thus the power of the first harmonic, $|h| = 1$.

Results

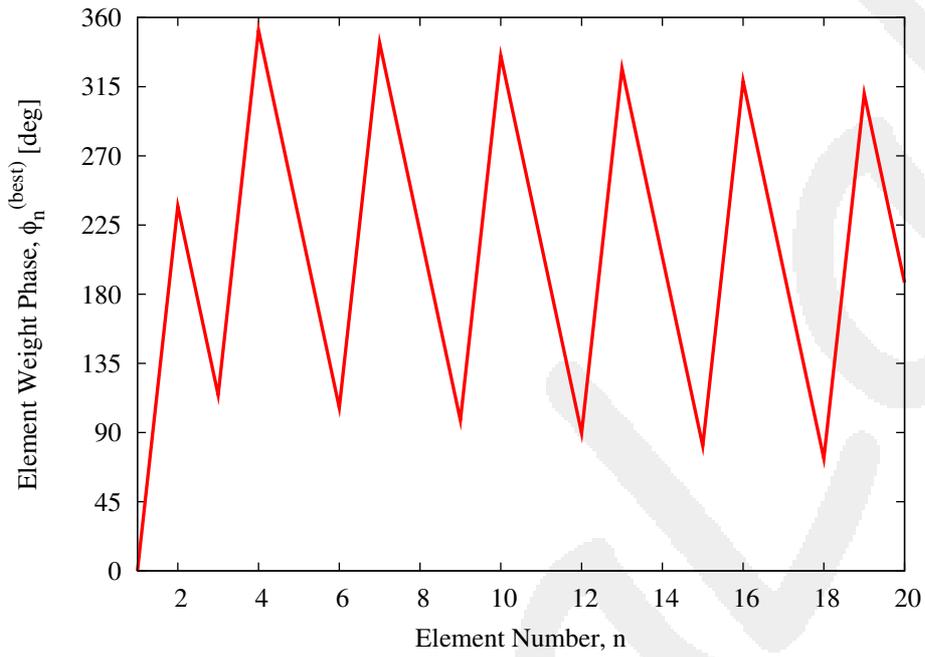


Fig.10 - Element Weight Phase

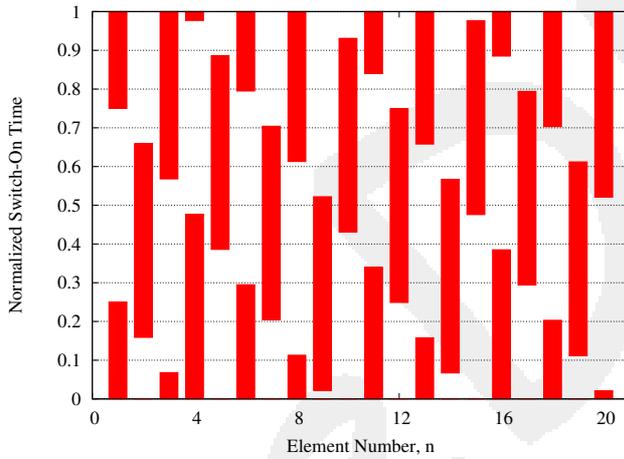


Fig.11 - Initial Pulse Sequence

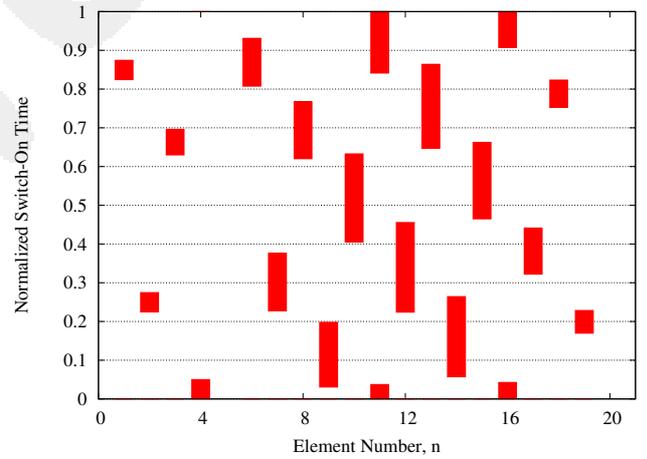


Fig.12 - Optimized Pulse Sequence

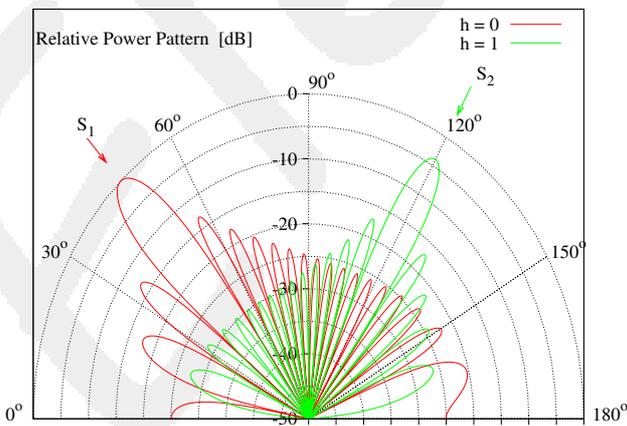


Fig.13 - Initial Pattern

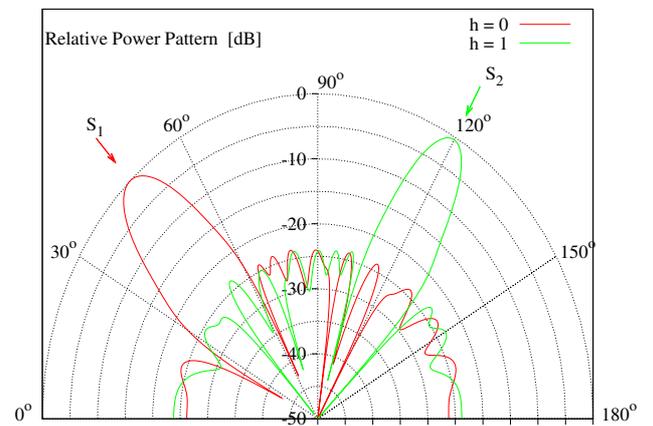


Fig.14 - Optimized Pattern

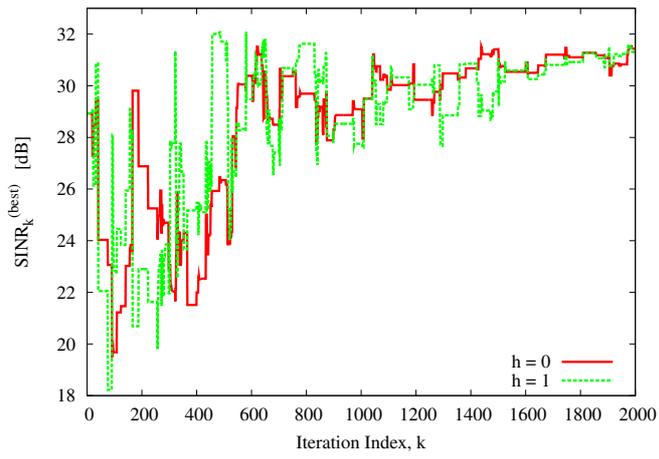


Fig.15 - SINR

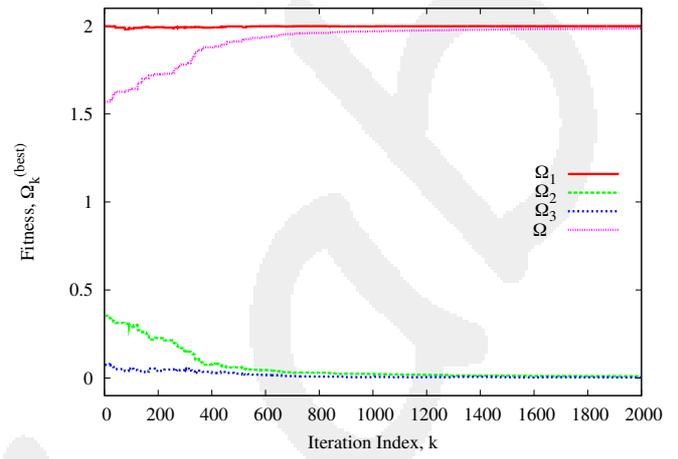


Fig.16 - Fitness

TEST CASE 2.b - Two Signals and $\theta_{(h=0)} \neq 90^\circ$

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 50^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta = 30^\circ$
- Initial Pulse width⁴: $\tau_n = 0.5$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 1$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

⁴ $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = 0.5$ maximizes the array factor, thus the power of the first harmonic, $|h| = 1$.

Results

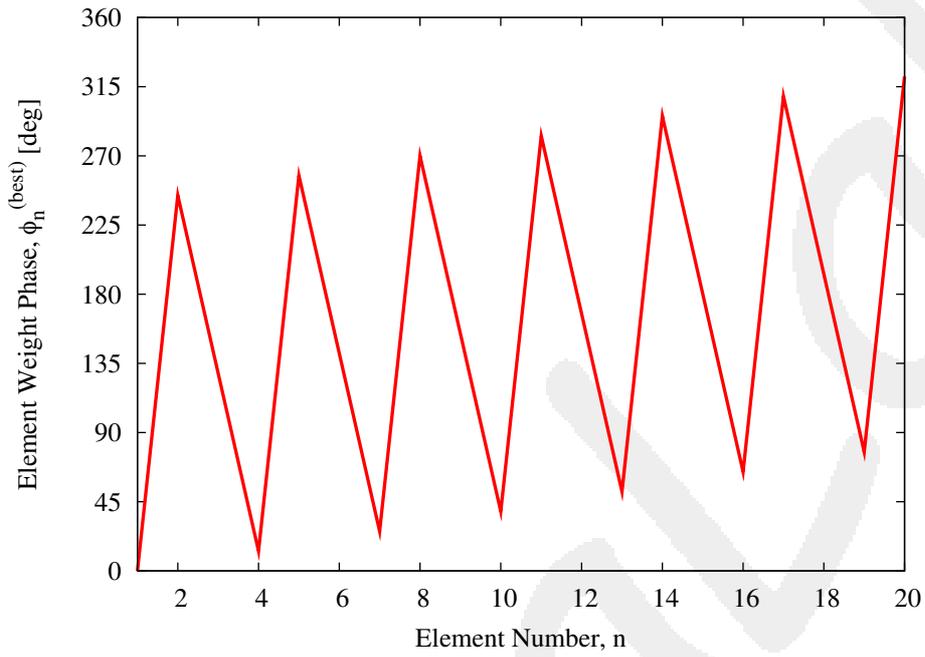


Fig.17 - Element Weight Phase

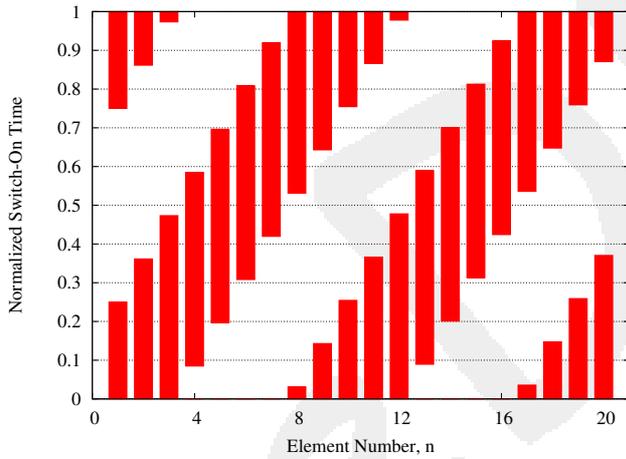


Fig.18 - Initial Pulse Sequence

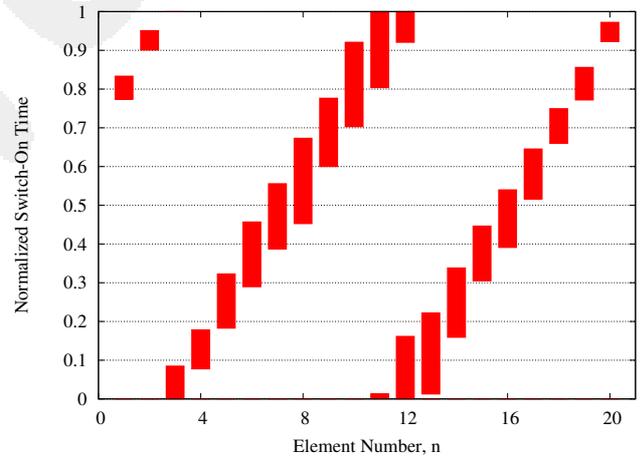


Fig.19 - Optimized Pulse Sequence

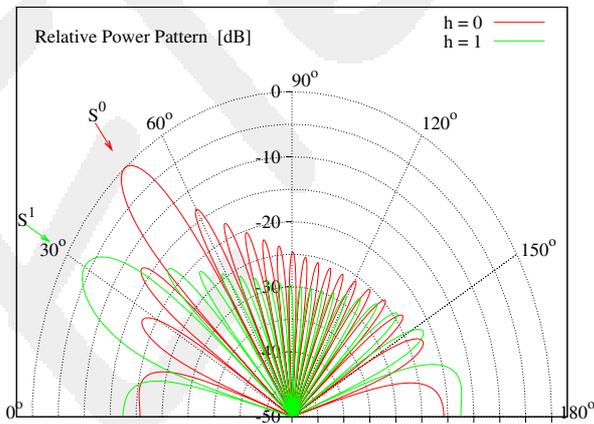


Fig.20 - Initial Pattern

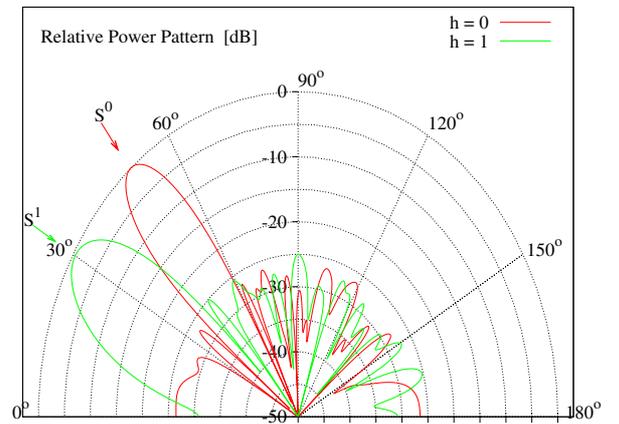


Fig.21 - Optimized Pattern

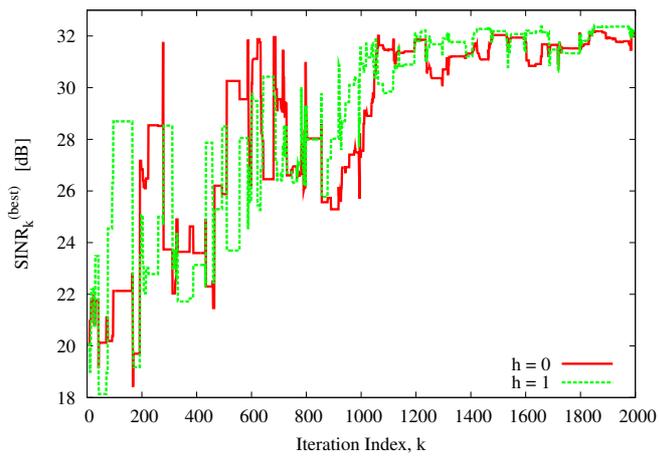


Fig.22 - SINR

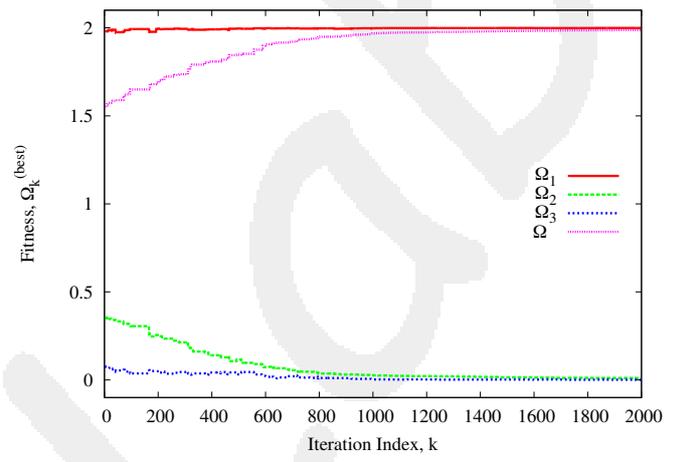


Fig.23 - Fitness

TEST CASE 3 - Three Signals

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 90^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta = 115.66^\circ$
- Signal $i = 2$, desired in the harmonic frequency, $h = 2$ with DOA: $\theta = 150^\circ$
- Uniform phase weighting of elements: $\varphi_n = 0$
- Initial Pulse width⁵: $\tau_n = 0.25$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 2$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

⁵ $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = 0.25$ maximizes the array factor, thus the power of the first harmonic, $|h| = 2$.

Results

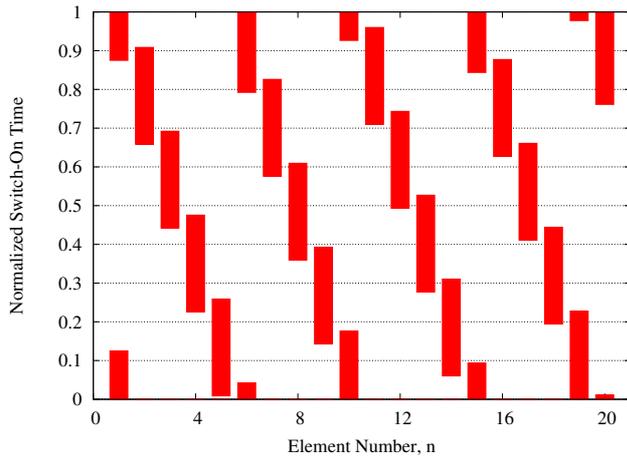


Fig.24 - Initial Pulse Sequence

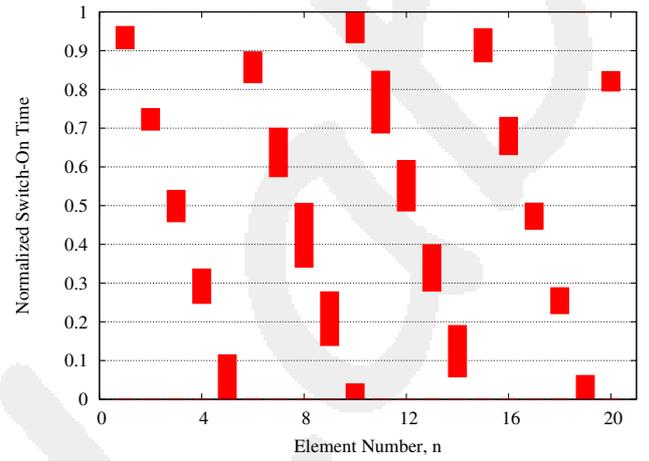


Fig.25 - Optimized Pulse Sequence

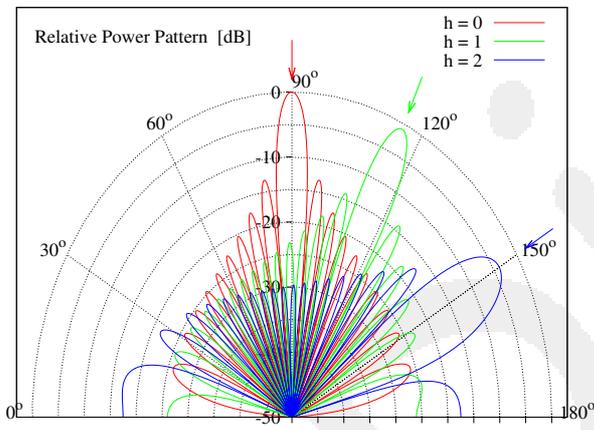


Fig.26 - Initial Pattern

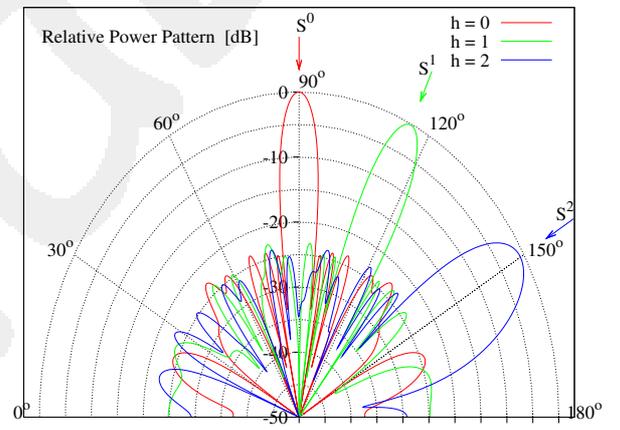


Fig.27 - Optimized Pattern

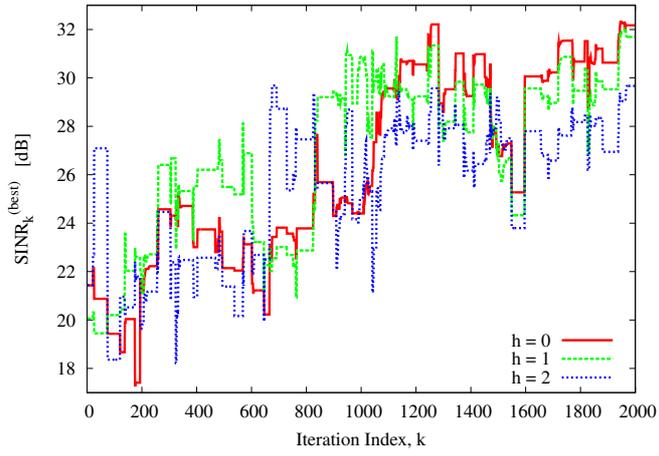


Fig.28 - SINR

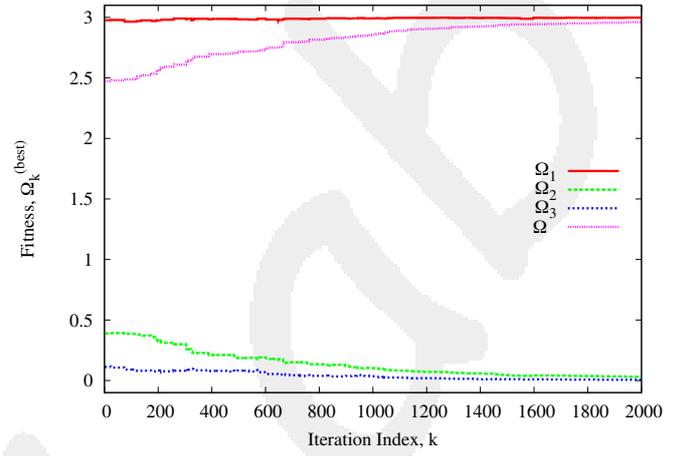


Fig.29 - Fitness

TEST CASE 4 - Four Signals

Test Case Description

- Signal $i = 0$, desired in the fundamental frequency, $h = 0$, with DOA: $\theta = 90^\circ$
- Signal $i = 1$, desired in the harmonic frequency, $h = 1$ with DOA: $\theta = 65.8^\circ$
- Signal $i = 2$, desired in the harmonic frequency, $h = 2$ with DOA: $\theta = 35^\circ$
- Signal $i = 3$, desired in the harmonic frequency, $h = 3$ with DOA: $\theta = 140.5^\circ$
- Uniform phase weighting of elements: $\varphi_n = 0$
- Initial Pulse width⁶: $\tau_n = \frac{1}{6}$
- Initial Pulse shift: $i_n^r = \frac{\varphi_n}{2\pi h} + \frac{\beta z_n \cos(\theta^i)}{2\pi h} - \frac{\tau_n}{2} - \frac{m}{h}$

where:

- φ_n phase of the array element weights
- z_n is the z coordinate of the elements.
- $h = 2$ is the harmonic index in which signal i will be received, and θ^i is the DOA of this signal. The initial pattern in the h^{th} harmonic will be directed to this angle.
- m is an integer chosen such that the constraint, $0 \leq i_n^r \leq 1$, is fulfilled.

⁶ $F^{(h)}(\theta) = \sum_{n=0}^{N-1} A_n U_{hn} e^{j\beta z_n \cos(\theta)}$

$U_{hn} = \tau_n \text{sinc}(h\pi\tau_n) e^{-jh\pi(2i_n^r + \tau_n)}$.

The value $\tau_n = \frac{1}{6}$ maximizes the array factor, thus the power of the harmonic, $|h| = 3$.

Results

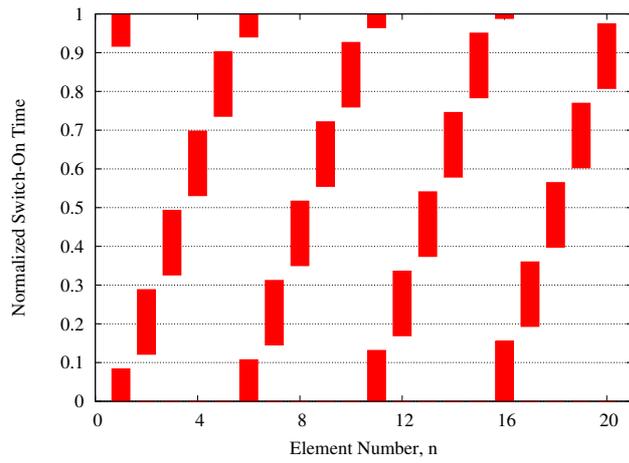


Fig.30 - Initial Pulse Sequence

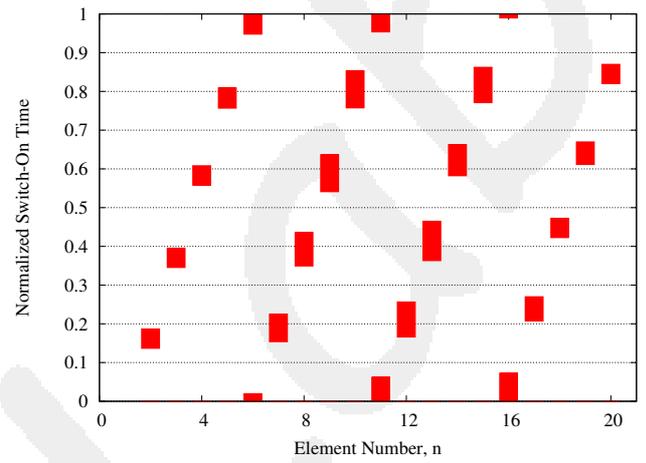


Fig.31 - Optimized Pulse Sequence

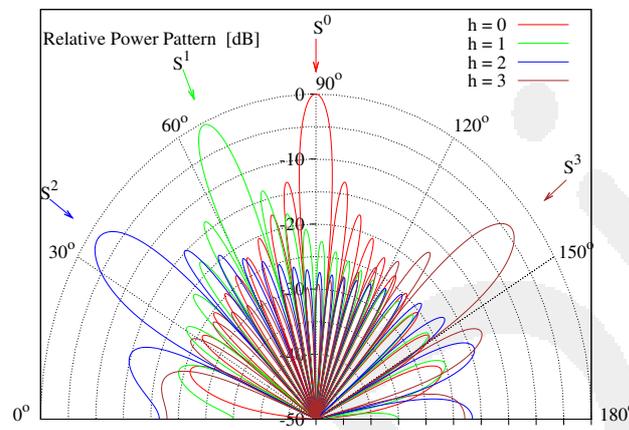


Fig.32 - Initial Pattern

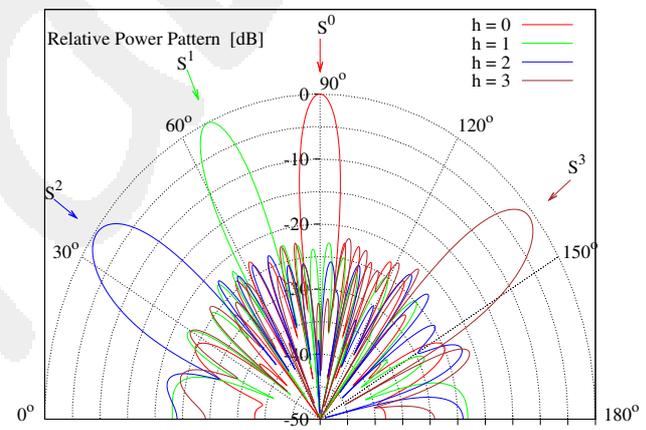


Fig.33 - Optimized Pattern

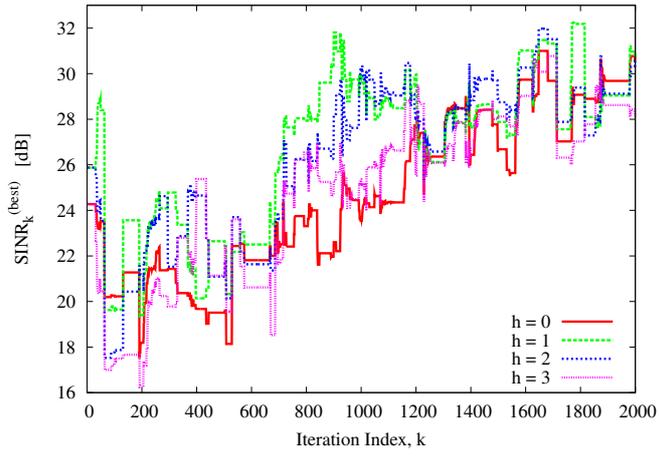


Fig.34 - SINR

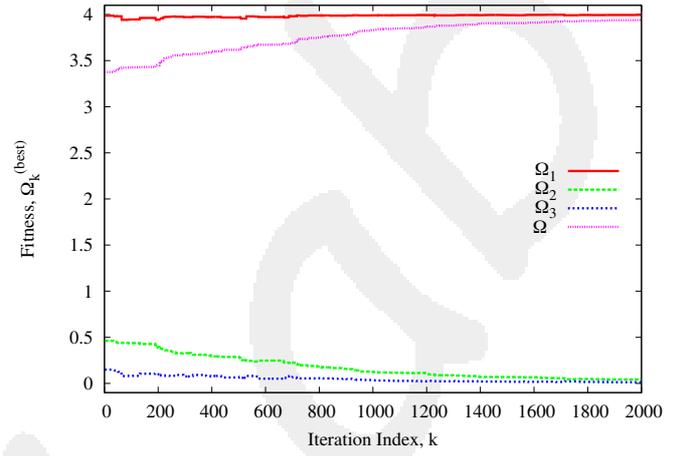


Fig.35 - Fitness

Two Signals

	$h = 0$		$h = 1$	
	Initial	Optimized	Initial	Optimized
$P(61^\circ)$ [dB]	-27.23	-38.59	-3.92	-0.5
$P(90^\circ)$ [dB]	0	0	-31.01	-52.14
SLL [dB]	-13.19	-21.7	-17.11	-21.6
$SINR$ [dB]	26.37	30.88	26.37	31.58

Table I: SLL, Null Depth, and SINR for pattern with two signals

Two Signals - Non Broad Side

	$h = 0$		$h = 1$	
	Initial	Optimized	Initial	Optimized
$P(47^\circ)$ [dB]	0	0	-34.86	-42.40
$P(120^\circ)$ [dB]	-30.87	-38.29	-3.92	-0.46
SLL [dB]	-13.19	-23.62	-17.11	-23.23
$SINR$ [dB]	28.93	31.43	28.93	31.31

Table I: SLL, Null Depth, and SINR for pattern with two signals Non-Broad-Side

Three Signals

	$h = 0$		$h = 1$		$h = 2$	
	Initial	Optimized	Initial	Optimized	Initial	Optimized
$P(90^\circ)$ [dB]	0	0	-24.08	-48.27	-31.04	-34.50
$P(116^\circ)$ [dB]	-23.28	-51.13	-0.91	-0.20	-27.23	-40.79
$P(150^\circ)$ [dB]	-26.95	-44.33	-24.23	-39.82	-3.92	-0.80
SLL [dB]	-13.19	-22.85	-14.10	-22.80	-17.11	-22.44
$SINR$ [dB]	21.43	32.17	20.06	31.69	21.43	29.67

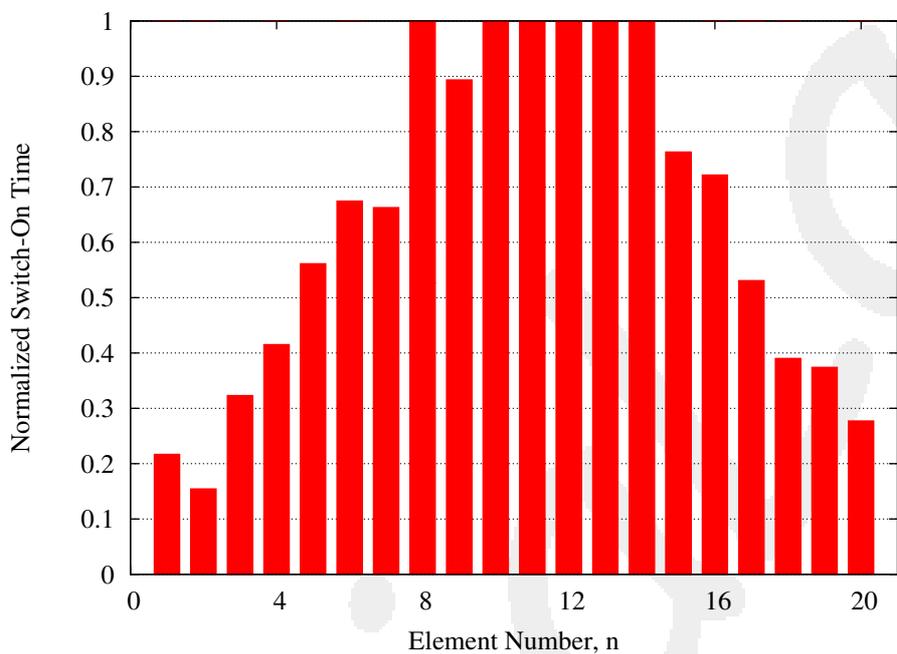
Table I: SLL, Null Depth, and SINR for pattern with three signals

Four Signals

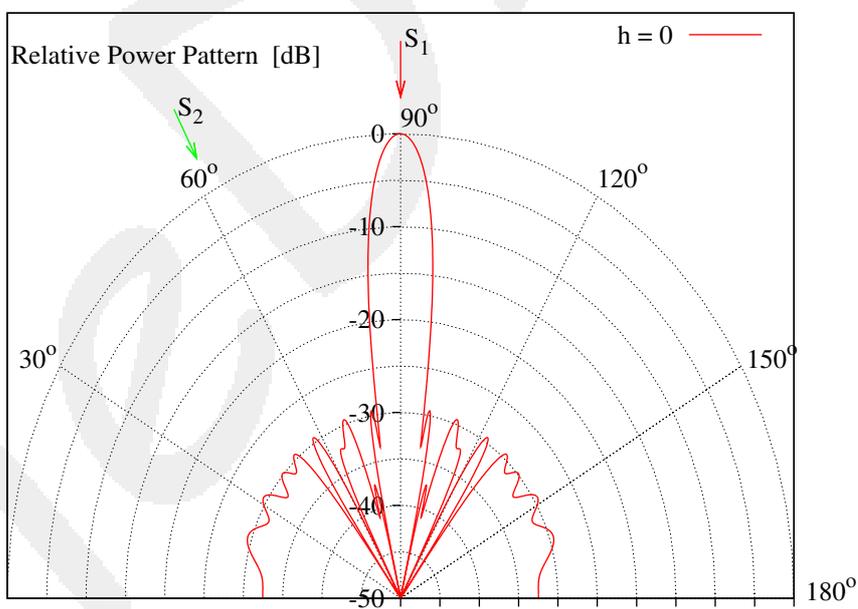
	$h = 0$		$h = 1$		$h = 2$		$h = 3$	
	Initial	Optimized	Initial	Optimized	Initial	Optimized	Initial	Optimized
$P(35^\circ)$ [dB]	-30.68	-39.09	-32.71	-42.93	-1.65	-0.39	-35.90	-35.41
$P(65^\circ)$ [dB]	-32.35	-43.00	-0.40	-0.097	-33.60	-38.15	-34.44	-42.28
$P(90^\circ)$ [dB]	0	0	-33.34	-39.89	-32.61	-42.67	-31.67	-33.26
$P(140^\circ)$ [dB]	-27.57	-38.63	-31.05	-38.33	-33.89	-41.48	-3.92	-0.89
SLL [dB]	-13.19	-22.56	-13.59	-22.27	-14.84	-21.70	-17.11	-20.86
$SINR$ [dB]	24.27	30.59	25.87	30.71	25.87	30.57	24.28	28.06

Table I: SLL and Null Depth for pattern with four signals

Two Signals - Optimization only on $h = 0$



(a)



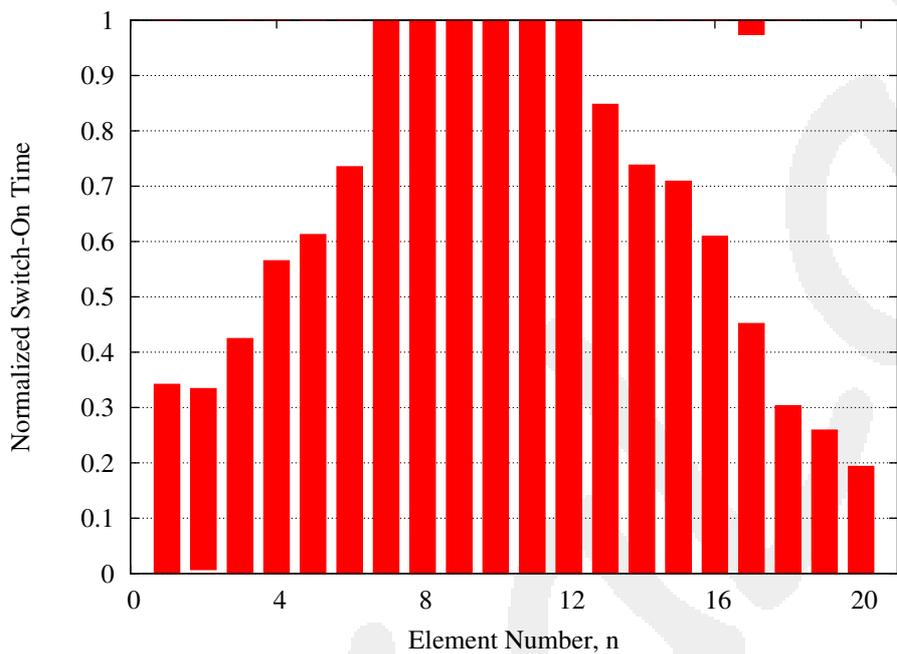
(b)

Figure. xxx., "Pulse Sequences and Pattern"

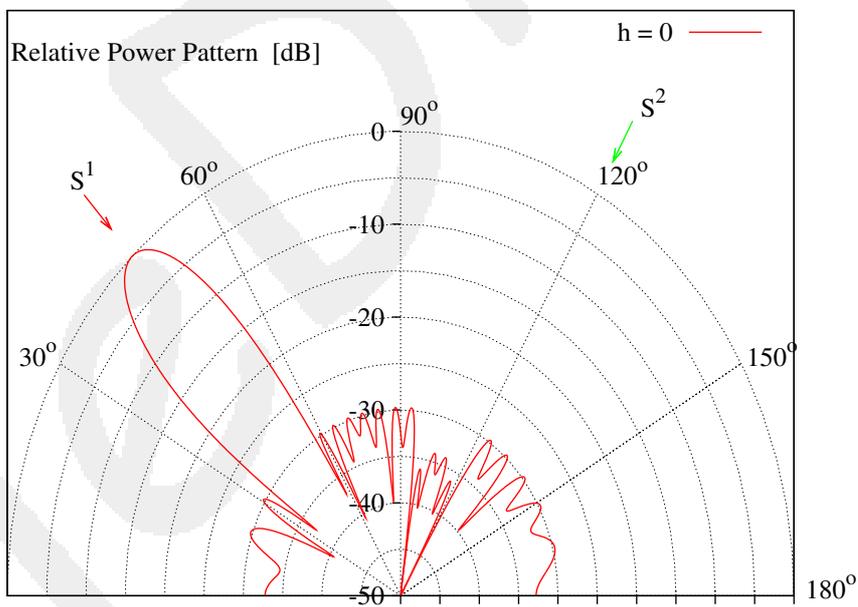
	$h = 0$	
	Initial	Optimized
$P(61^\circ)$ [dB]	-27.23	-65.15
$P(90^\circ)$ [dB]	0	0
SLL [dB]	-13.19	-29.48
$SINR$ [dB]	26.37	32.20

Table I: SLL, Null Depth, and SINR for pattern with One Signal One Interferer

Two Signals - Non Broad Side - Optimization only on $h = 0$



(a)



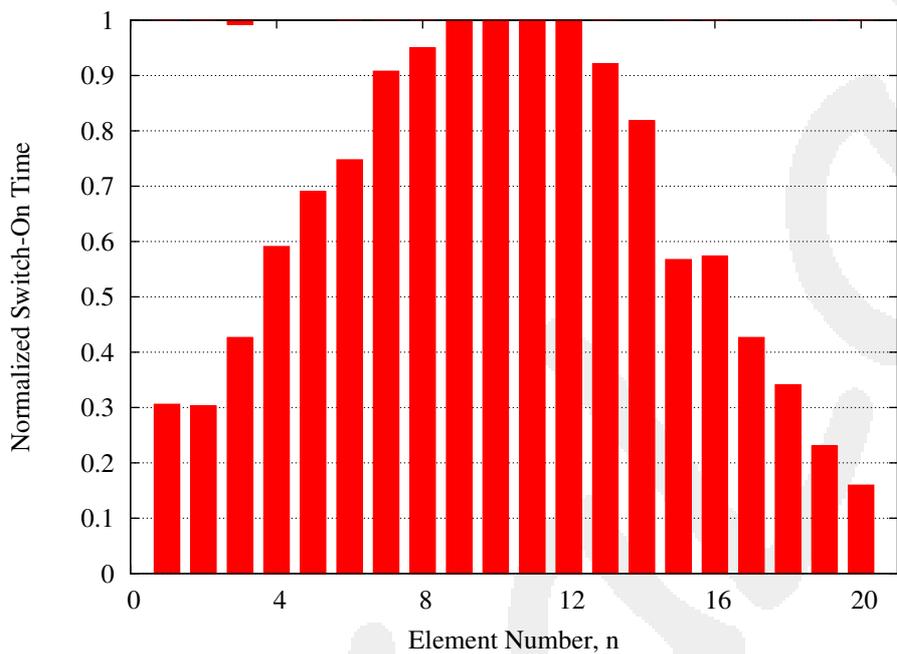
(b)

Figure. xxx., "Pulse Sequences and Pattern"

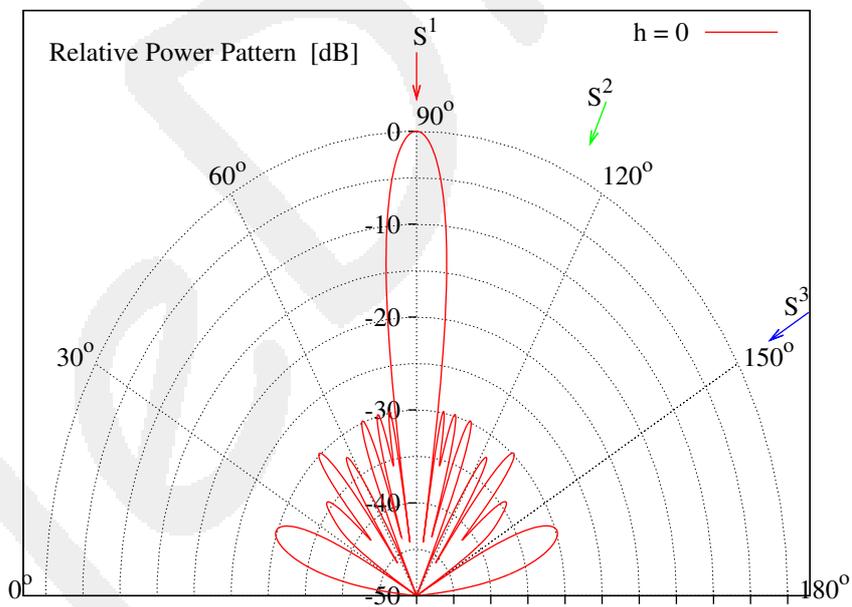
	$h = 0$	
	Initial	Optimized
$P(47^\circ)$ [dB]	0	0
$P(120^\circ)$ [dB]	-30.87	-65.77
SLL [dB]	-13.19	-29.73
$SINR$ [dB]	28.93	32.29

Table I: SLL, Null Depth, and SINR for pattern with One Signal One Interferer Non-Broad-Side

Three Signals - Optimization only on $h = 0$



(a)



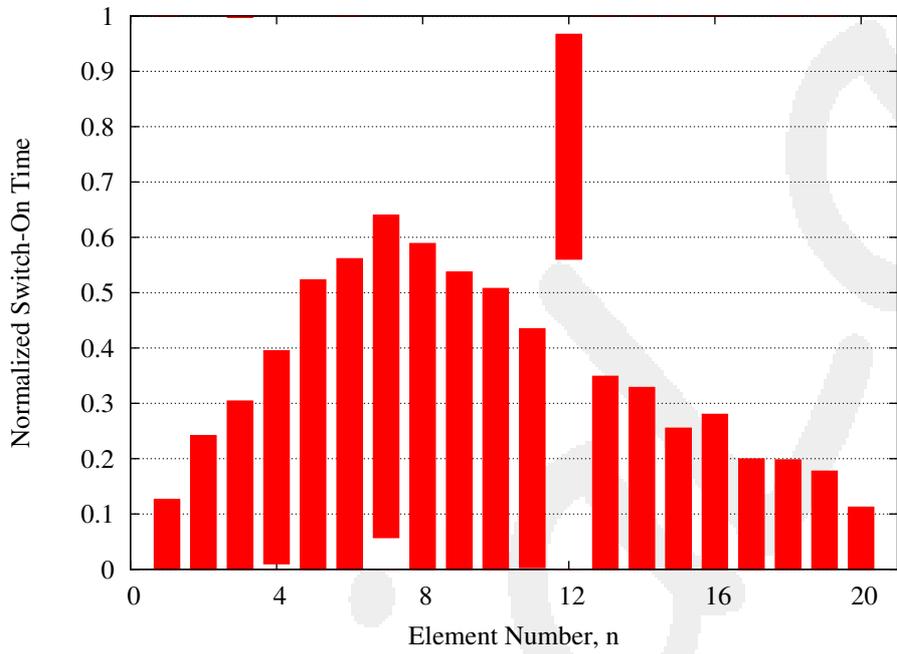
(b)

Figure. xxx., "Pulse Sequences and Pattern"

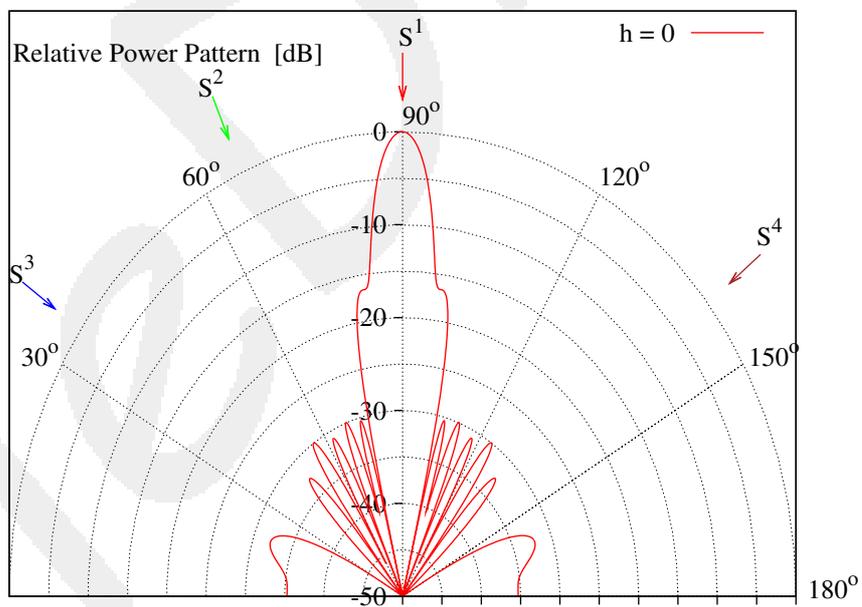
	$h = 0$	
	Initial	Optimized
$P(90^\circ)$ [dB]	0	0
$P(116^\circ)$ [dB]	-23.28	-69.22
$P(150^\circ)$ [dB]	-26.95	-73.89
SLL [dB]	-13.19	-29.82
$SINR$ [dB]	21.43	32.24

Table I: SLL, Null Depth, and SINR for pattern with One Signal and two interferers

Four Signals - Optimization only on $h = 0$



(a)



(b)

Figure. *xxx.*, "Pulse Sequences and Pattern"

	$h = 0$	
	Initial	Optimized
$P(35^\circ)$ [dB]	-30.68	-74.24
$P(65^\circ)$ [dB]	-32.35	-78.71
$P(90^\circ)$ [dB]	0	0
$P(140^\circ)$ [dB]	-27.57	-69.54
SLL [dB]	-13.19	-29.98
$SINR$ [dB]	24.27	32.27

Table I: SLL and Null Depth for pattern with One Signal and three interferers

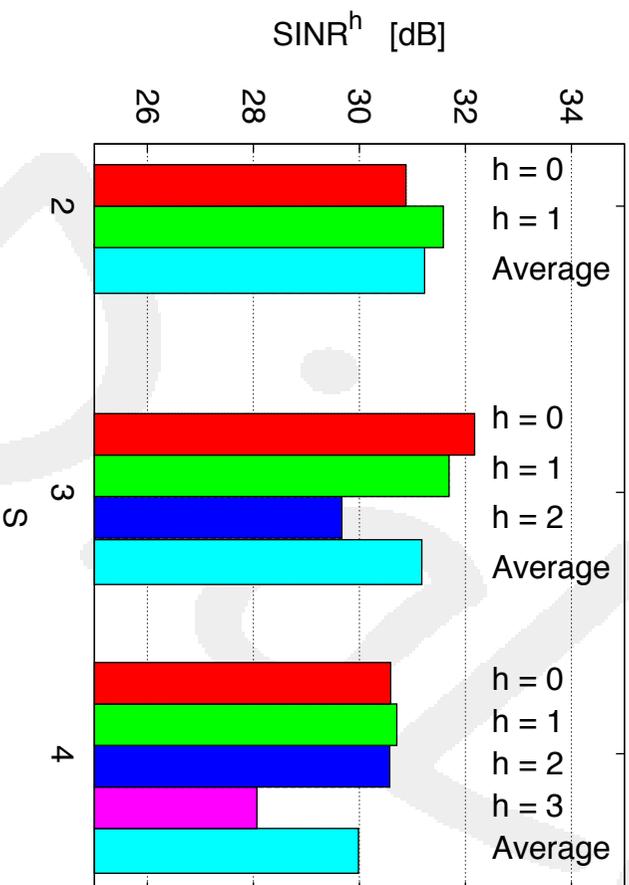


Fig. xxx, "SINR Comparison."

Additional Data:

Average of Time per time step = 738.385sec.

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