Sideband Suppression in Time-Modulated Linear Arrays via Element Factor

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Abstract

The reduction of the power losses in the sideband radiation of directive time-modulated linear arrays is addressed in this report. The approach is based on the optimization of the pulse sequence modulating the static excitations of the array elements to exploit pattern multiplication effects such that the sideband level of the harmonic pattern term related to the array factor is balanced by the element pattern distribution. A preliminary assessment is proposed in this report in order to show the effectiveness of the approach to suppress both the sideband level or the amount of sideband radiation.

Definitions

Array Factor

Let us consider a linear array of N isotropic elements aligned along the z-axis. The array factor with the $e^{j\omega_o t}$ term explicitly indicated is:

$$AF(\theta, t) = e^{j\omega_0 t} \sum_{n=1}^{N} \alpha_n e^{j(n-1)kd\cos\theta} \quad \theta \in [0; \pi] \tag{1}$$

where α_n is the complex excitation of the n-th element of the array, d is the element spacing, and $\omega_o = 2\pi f_0$ is the RF angular frequency of the carrier signal.

Time-Modulation technique applied in the array synthesis can be realized using in the feed network a set of RF switches connected to the elements of the array, to turn-on/off the state of the elements in specific intervals of time: the array factor becomes

$$AF(\theta, t) = e^{j\omega_0 t} \sum_{n=1}^{N} \alpha_n U_n(t)\, e^{j(n-1)kd\cos\theta} \tag{2}$$

where $U_n(t)$ is the *time switching* periodic function of period T_p related to the n-th element, usually of rectangular impulsive kind:

$$U_n(t) = \begin{cases} 1, & t'_n \leq t \leq t''_n \\ 0, & elsewhere \end{cases} \tag{3}$$

where $0 \leq t'_n \leq t''_n \leq T_p$.

Expanding the switching function $U_n(t)$ in the Fourier serier:

$$U_n(t) = \sum_{h=-\infty}^{+\infty} u_{hn} e^{jh\omega_p t} \tag{4}$$

where

$$u_{hn} = \frac{1}{\pi h}\left[\sin\left(\frac{h\omega_p}{2}(t''_n - t'_n)\right)\right] e^{-j\frac{h\omega_p}{2}(t''_n + t'_n)} = \tau_n \left[\mathrm{sinc}\,(h\pi\tau_n)\right] e^{-jh\pi(\tau_n + 2\tau'_n)} \tag{5}$$

where $\tau_n = (t''_n - t'_n)/T_p$, $\tau'_n = t'_n/T_p$ and $\tau''_n = t''_n/T_p$.

The array factor expressed in (2) becomes

$$AF(\theta, t) = \sum_{h=-\infty}^{+\infty} AF_h(\theta, t) \tag{6}$$

where

$$AF_h(\theta, t) = e^{j(\omega_0 + h\omega_p)t} \sum_{n=1}^{N} a_{hn} e^{j(n-1)kd\cos\theta} \tag{7}$$

where $a_{hn} = \alpha_n u_{hn}$.

The electric field radiated by the array is:

$$E(\theta, t) = \sum_{h=-\infty}^{+\infty} E_h(\theta, t) \tag{8}$$

where the harmonic components of the electric field are

$$|E_h(\theta,t)| = e_0(\theta)\left|\sum_{n=1}^{N} a_{hn} e^{j(n-1)kd\cos\theta}\right| \tag{9}$$

where $e_0(\theta)$ is the element factor.

The element factor of a short dipole aligned along the z-axis is defined [1]:

$$e_0(\theta) = \sin(\theta) \tag{10}$$

where l is the length of the dipole (for hypothesis $l \leq 0.01\lambda$).

$-3\,[dB]$ **Main Beamwidth** BW

$$BW(t) = \theta_A(t) - \theta_B(t) \tag{11}$$

where $\theta_A(t)$ and $\theta_B(\mathrm{t})$ are the angular positions such that $|AF(\theta,t)| = -3\,[dB]$.

Sidelobe Level (SLL)

The maximum level of the grating lobes.

Sideband Level (SBL)

The maximum level of the sideband radiations.

Total Power and Sideband Radiation

The total power radiated by a TMLA is defined as:

$$P_{tot} = \sum_{n=0}^{N-1}\left\{|\alpha_n|^2 \sum_{h=-\infty}^{\infty} u_{hn}^2\right\} + 2\sum_{m,\,n\,\in\,\Omega_{mn}}\left\{\mathrm{Re}\left\{\alpha_m\alpha_n^*\right\}\mathrm{sinc}\left[kd(m-n)\right]\sum_{h=-\infty}^{\infty} u_{hn}u_{hm}\right\} \tag{12}$$

where u_{hn} is the h-th expansion coefficient of the Fourier series of the modulating function related to the n-th element, and Ω_{mn} represents the set of the indexes that correspond to all nonrepeated (m,n) pairs with $m \neq n$. The (12) can be written as a summation of infinite terms representing the power of all the frequencies, central and harmonics:

$$P_{tot} = \sum_{h=-\infty}^{\infty} P_h \tag{13}$$

The Sideband Radiation represents the amount of power radieted at the harmonic frequencies (usually expressed in percentage with respect to the total radiated power). A closed-form equation to calculate this power is exploited:

$$P_{SR} = \sum_{n=1}^{N}\left\{|\alpha_n|^2\,\tau_n(1-\tau_n)\right\} + \sum_{m,n\in\Omega_{mn}}\left\{\mathrm{Re}\left\{\alpha_m\alpha_n^*\right\}\mathrm{sinc}\left[k(z_m-z_n)\right]\left(\tau_{\{m,n\}_{MinVal}} - \tau_m\tau_n\right)\right\} \tag{14}$$

whereτ_n is the duration normalized with respect to the modulation period T_p of the pulse exciting the n-th element.

$$\tau_{\{m,n\}_{MinVal}} = \begin{cases} \tau_n & \text{if } \tau_n \leq \tau_m \\ \tau_m & \text{altrimenti} \end{cases} \tag{15}$$

Preliminary Results

TEST CASE 1

Goal

Sideband radiation minimization of a TMLA composed by real radiating elements (short dipoles considered in this test case) adopting the pulse shifting technique.

Test Case Description

- Number of Elements: $N = 16$
- Elements Spacing: $d = 0.5\lambda$
- Static Array Configuration: $\alpha_n = 1,\ n = 0, ..., N-1$
- Pattern at Central Frequency: $Dolph - Chebyshev\,,\, SLL = -30\,dB$
- Max Gain Pattern Direction : $\theta^{max} = 90°$

1.a) Optimization Approach: PS-PSO, SBL Min.

The optimization process though the PSO algorithm acts just on the temporal shift of the pulses;

- Number of Variables: $X = 16$
- Number of Particles: $S = 30$
- Number of Iterations: $I = 200$
- Inertial Weight: Linearly varying: $0.9\,to\,0.4$
- Cost Function: $(H = 2)$

$$\Psi^{PSO}\left[\tau'_n(i_k)\right] = \sum_{h=1}^{H} SBL_h^{act,(i_k)} \tag{16}$$

1.b) Optimization Approach: PS-PSO, SR Min.

The optimization process though the PSO algorithm acts just on the temporal shift of the pulses;

- Number of Variables: $X = 16$
- Number of Particles: $S = 30$
- Number of Iterations: $I = 1000$
- Inertial Weight: Linearly varying: $0.9\,to\,0.4$
- Cost Function:

$$\Psi^{PSO}\left[\tau'_n(i_k)\right] = P_{SR}^{act,(i_k)} \tag{17}$$

Dolph-Chebyshev Pattern, SLL=-30 dB - End-Fire Harmonic Pattern h=1

The pulse sequence configuration proposed in the following to generate the End-Fire radiation pattern at the harmonic $h = 1$ presents a very high amount of Sideband Radiation, due to the value of many pulses close to the half duration of the modulation period.

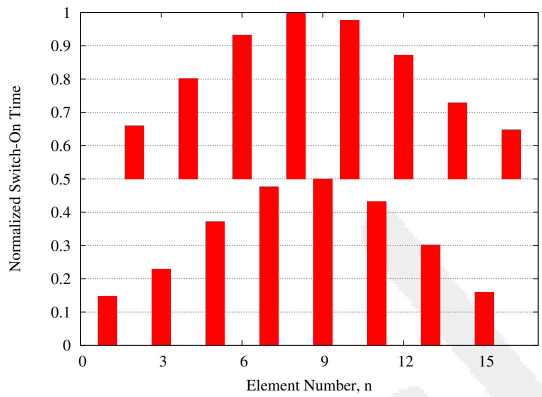

Fig.1 - Pulse Sequence [5]

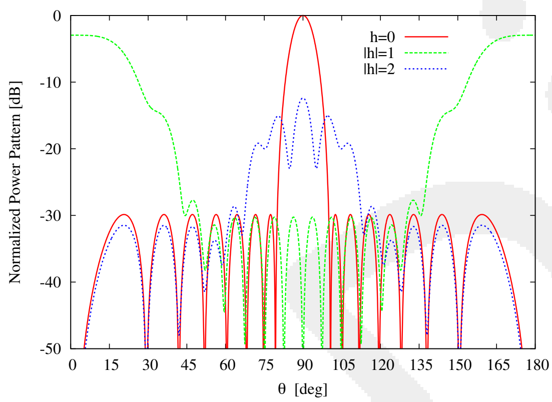

Fig.2 - Pattern(θ)

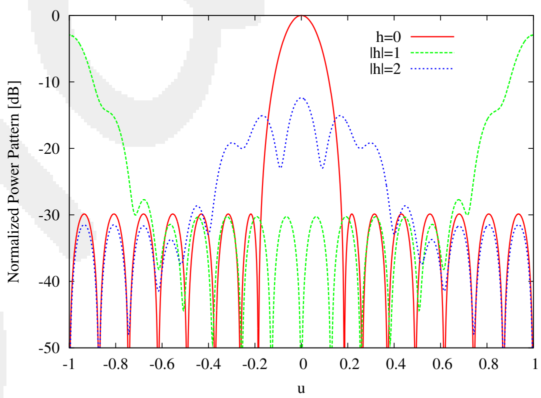

Fig.3 - Pattern(u)

The normalization of the pulse duration with respect to the modulation period, allows to reduce the SR from 62.07% to 24.16%.

It is possible to demonstrate such a normalization is always required to minimize the percentage of sideband radiation with respect to the total radiated power (Appendix A).

Dolph-Chebyshev Pattern, SLL=-30 dB - End-Fire Harmonic Pattern h=1 - Normalized Pulse Sequence

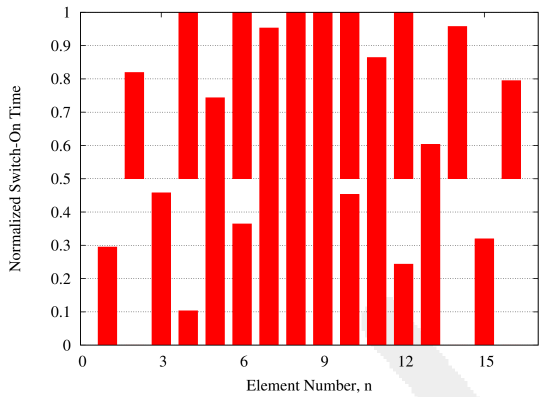

Fig.4 - Pulse Sequence

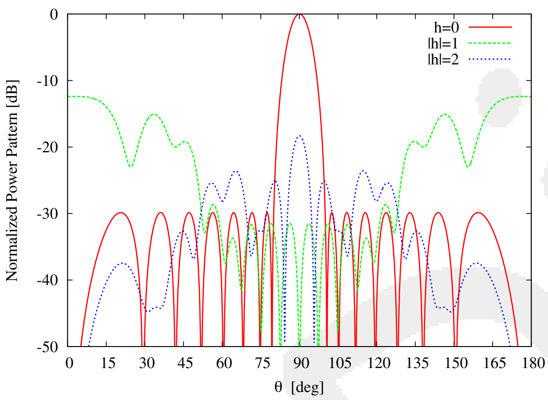

Fig.5 - Pattern(θ) - Ideal

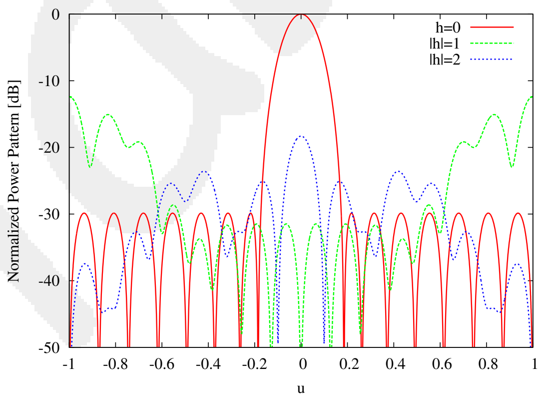

Fig.6 - Pattern(u) - Ideal

Dolph-Chebyshev Pattern, SLL=-30 dB - End-Fire Harmonic Pattern h=1 - Short Dipole Elements Array

Let us consider an array of short dipole elements positioned and aligned along the z-axis.

The pulse sequence configuration showed in Fig.4, applied to an array composed by short dipole, allows to translate in end-fire the radiated pattern at the first harmonic $h = 1$ (see Appendix B for demonstration):

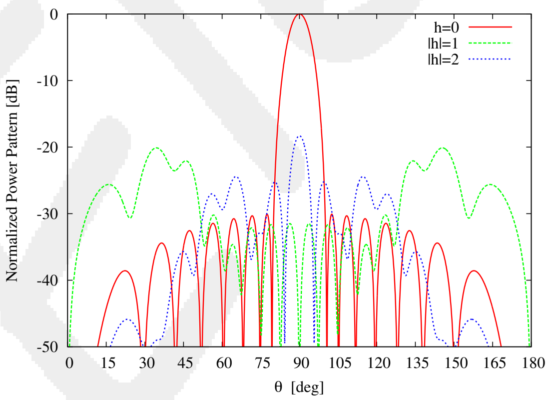

Fig.7 - Pattern(θ) - Dipole

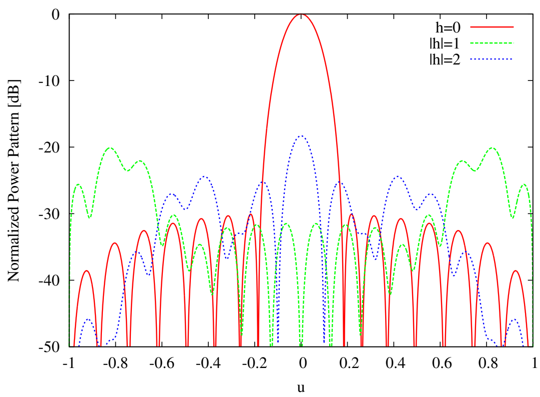

Fig.8 - Pattern(u) - Dipole

It is possible to see in Fig. 7-8 that part of the radiated power is suppressed thanks to the element factor of the dipole. The SBL at the first harmonic $h = 1$ is reduced from $-12.39\,dB$ to $-20.12\,dB$ but the SBL at the second harmonic remain $-18.30\,dB$. The sideband radiation is also reduced from 22.50% to 12.18%.

Dolph-Chebyshev Pattern, SLL=-30 dB - PS-PSO, SBL Min. - Short Dipole Elements Array

The PSO can be used to optimize the pulse sequence configuration minimizing the SBL.

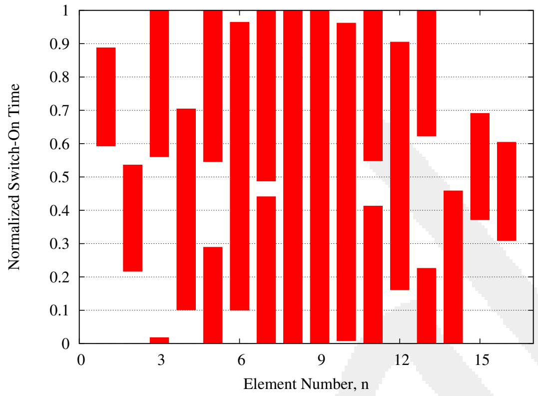

Fig.9 - Pulse Sequence

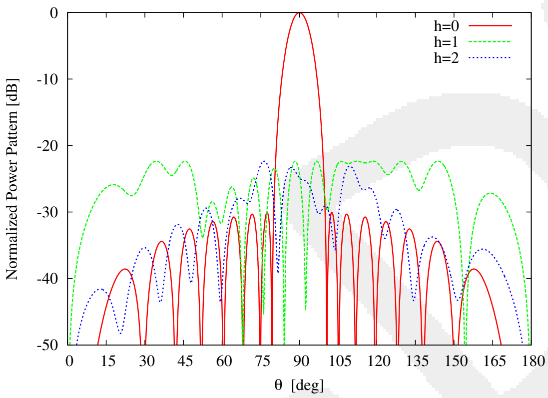

Fig.10 - Pattern(θ)

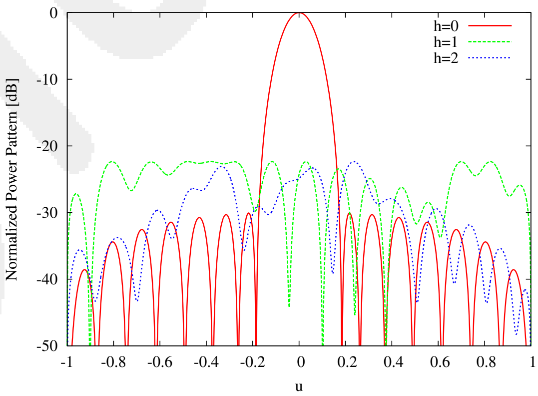

Fig.11 - Pattern(u)

The SBL is reduced with respect to the previous configuration from $-18.30\,dB$ to $-22.36\,dB$.

Dolph-Chebyshev Pattern, SLL=-30 dB - PS-PSO, SR Min. - Short Dipole Elements Array

The PSO can be used also to optimize the pulse sequence configuration minimizing SR rather than SBL.

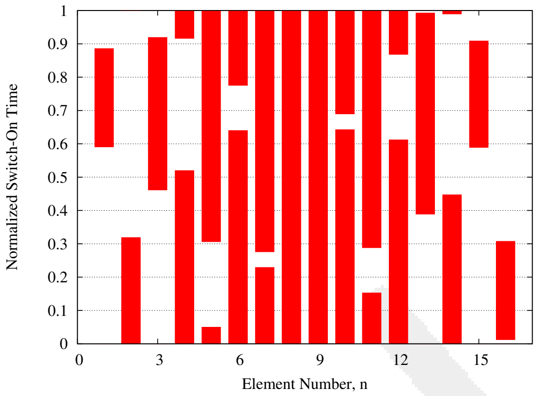

Fig.12 - Pulse Sequence

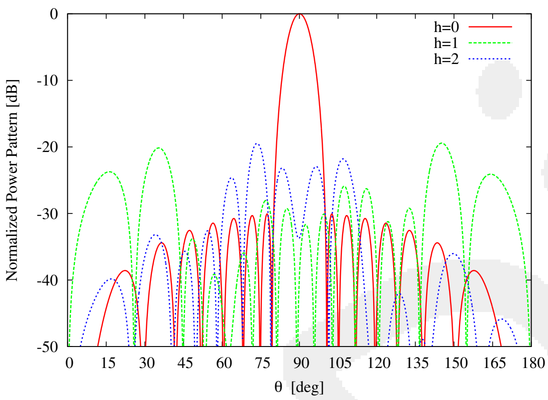

Fig.13 - Pattern(θ)

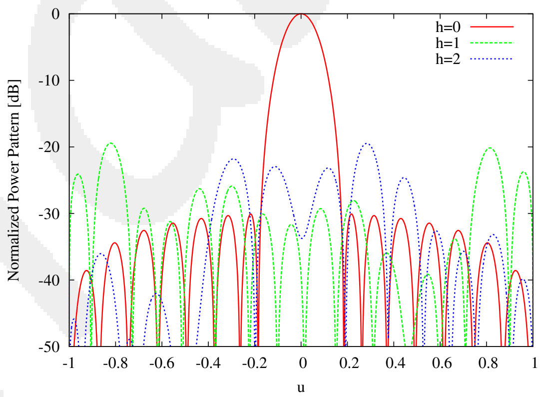

Fig.14 - Pattern(u)

The SR is reduced with respect to the configuration proposed in Fig.4 from 12.18% to 11.45%.

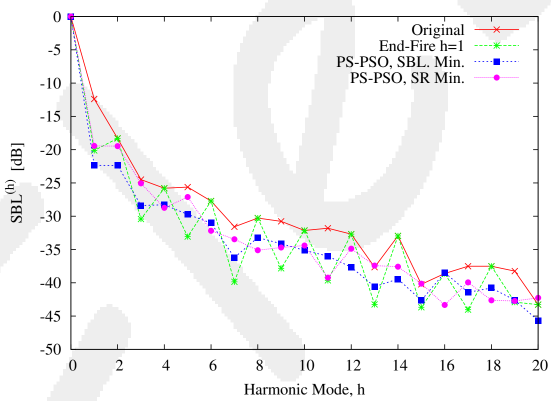

Fig.15 - Harmonics SBL

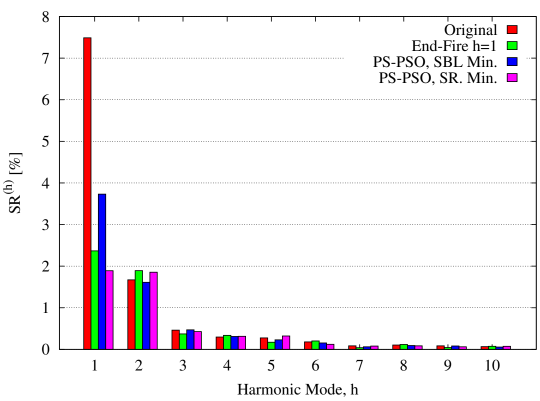

Fig.16 - Harmonics SR

	$SLL\,[dB]$	$SBL\,[dB]$	$SBL_1\,[dB]$	$SBL_2\,[dB]$	$BW\,[deg]$	$SR\,[\%]$
$Dolph-Chebyshev$	-30.08	-12.39	-12.39	-18.30	15.85	22.50
$End\,Fire\,h=1\,Pattern$	-30.08	-18.30	-20.12	-18.30	15.85	12.18
$PS-PSO,\,SBL\,Min.$	-30.08	-22.36	-22.36	-22.36	15.85	14.58
$PS-PSO,\,SR\,Min.$	-30.08	-19.44	-19.44	-19.47	15.85	11.45

Tab.1 - Sidelobe Lebel (SLL), Sideband Level (SBL), -3 dB Beamwidth (BW), Sideband Radiation (SR).

Appendix A

Normalization of the Pulse Durations wrt. the Modulation Period

The proof reported below shows the convenience in applying the normalization of the pulse duration with respect to the modulation period to reduce the amount of sideband radiation with respect to the total radiated power.

Considering a TMLA of N elements, we assume (this hyphotesis is not relevant to the result of the demonstration) that the element associated at the pulse with the longest duration is the first element of the array:

$$x = \tau_1 \tag{18}$$

We want to demonstrate that the amount of the sideband radiation with respect to the total radiated power is minimum when $x = 1$, that is when the longest pulse duration in the pulses sequence is equal to the modulation period.

We can write all the others pulse durations as functions of x:

$$\tau_n = x\gamma_n \quad n = 2, ..., N \tag{19}$$

where $\gamma_n = \tau_n/\tau_1$.

The sideband radiation is defined:

$$P_{SR} = \sum_{n=1}^{N} \left\{ |\alpha_n|^2 \tau_n(1-\tau_n) \right\} + 2 \sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \left(\tau_{\{m,n\}_{MinVal}} - \tau_m\tau_n\right) \right\} \tag{20}$$

The total radiated power is calculated as:

$$P_{TOT} = \sum_{n=1}^{N} \left\{ |\alpha_n|^2 \tau_n(1-\tau_n) \right\} + 2 \sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \left(\tau_{\{m,n\}_{MinVal}} - \tau_m\tau_n\right) \right\} +$$

$$\sum_{n=1}^{N} \left\{ (|\alpha_n| \tau_n)^2 \right\} + 2 \sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \tau_m\tau_n \right\} \tag{21}$$

The ratio between the sideband radiation and the total radiated power can be written as:

$$\eta = \frac{P_{SR}}{P_{TOT}} = \frac{\sum_{n=1}^{N} \left\{ |\alpha_n|^2 \tau_n(1-\tau_n) \right\} + 2\sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \left(\tau_{\{m,n\}_{MinVal}} - \tau_m\tau_n\right) \right\}}{\sum_{n=1}^{N} \left\{ (|\alpha_n|^2 \tau_n) \right\} + 2\sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \tau_{\{m,n\}_{MinVal}} \right\}} \tag{22}$$

Considering (19), the ratio (22) becomes:

$$\eta = 1 - x \frac{\sum_{n=1}^{N} \left\{ (|\alpha_n| \gamma_n)^2 \right\} + 2\sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \gamma_m\gamma_n \right\}}{\sum_{n=1}^{N} \left\{ (|\alpha_n|^2 \gamma_n) \right\} + 2\sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{\alpha_m \alpha_n^*\right\} \mathrm{sinc}\left[kd(m-n)\right] \gamma_{\{m,n\}_{MinVal}} \right\}} =$$

$$1 - x \frac{[A(\alpha_n, \gamma_n) + B(\alpha_n, \gamma_n)]}{[C(\alpha_n, \gamma_n) + D(\alpha_n, \gamma_n)]} = 1 - x\xi \tag{23}$$

where ξ is a constant coefficient, beacuse $\alpha_n, \gamma_n \quad n = 1, ..., N$ are constant values.

Since P_{SR} is a power, it cannot be a negative quantity, then:

$$C(\alpha_n, \gamma_n) - A(\alpha_n, \gamma_n) + D(\alpha_n, \gamma_n) - B(\alpha_n, \gamma_n) \geq 0 \tag{24}$$

where

$$A(\alpha_n, \gamma_n) = \sum_{n=1}^{N} \left\{ (|\alpha_n| \gamma_n)^2 \right\} \tag{25}$$

$$B(\alpha_n, \gamma_n) = 2 \sum_{m,n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{ \alpha_m \alpha_n^* \right\} \mathrm{sinc}\left[kd(m-n) \right] \gamma_m \gamma_n \right\} \tag{26}$$

$$C(\alpha_n, \gamma_n) = \sum_{n=1}^{N} \left\{ |\alpha_n|^2 \gamma_n \right\} \tag{27}$$

$$D(\alpha_n, \gamma_n) = 2 \sum_{m, n \in \Omega_{mn}} \left\{ \mathrm{Re}\left\{ \alpha_m \alpha_n^* \right\} \mathrm{sinc}\left[kd(m-n) \right] \gamma_{\{m,n\}_{MinVal}} \right\} \tag{28}$$

Rewriting the (24), we get

$$C(\alpha_n, \gamma_n) - A(\alpha_n, \gamma_n) \geq -D(\alpha_n, \gamma_n) + B(\alpha_n, \gamma_n) \tag{29}$$

and hence

$$C(\alpha_n, \gamma_n) + D(\alpha_n, \gamma_n) \geq A(\alpha_n, \gamma_n) + B(\alpha_n, \gamma_n) \tag{30}$$

Since $[A(\alpha_n, \gamma_n) + B(\alpha_n, \gamma_n)] \geq 0$ because it is the ratio between the power radiated at the central frequency and the term x^2, then $[C(\alpha_n, \gamma_n) + D(\alpha_n, \gamma_n)] \geq 0$ as well. Consequently, considering (23), it results $0 \leq \xi \leq 1$, and the value of x that minimize the equation (23) in the domain $(0, 1]$ is $x = 1$.

Appendix B

The array factor

$$AF_1(\theta) = \sum_{n=1}^{N} a_{hn} e^{j(n-1)\psi} = \sum_{n=1}^{N} a_{hn} e^{j(n-1)(kd\cos\theta+\beta)} = \sum_{n=1}^{N} a_{hn} e^{j(n-1)\beta} e^{j(n-1)(kd\cos\theta)} \quad \theta \in [0;\pi] \tag{31}$$

The maximum of the array factor occurs when

$$\psi = kd\cos\theta + \beta = 0 \tag{32}$$

If it is desired to have the maximum directed toward $\theta = 0$ then

$$\beta|_{\theta=0} = -kd \tag{33}$$

From equations (5) and (31) it is possible to see that the parameter τ'_n can be used to change the phase of the coefficients a_{1n}, with $n = 1, ..., N$ (the parameter τ_n remains inaltered: in such a way the pattern at the central frequency doesn't result modified)

$$e^{j(n-1)\,\beta|_{\theta=0}} = e^{-j(n-1)kd} = e^{-j\pi(2\tau'_n)}$$

$$(n-1)\,kd = \pi(2\tau'_n)$$

$$\tau'_n = (n-1)\,d_\lambda \quad n = 1, ..., N$$

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