Synthesis of Sparse Time-Modulated Linear Arrays with Minimum Power Losses

L. Poli, P. Rocca, A. Massa

Abstract

This report proposes a novel strategy aimed at synthesizing a sparse time-modulated array with the smallest number of elements affording a beam pattern close to a reference one at the carrier frequency, while minimizing the amount of sideband power under a user-defined threshold.

Contents

1 Mathematical Formulation

1.1 Sideband Radiation Computation

Consider an array of N identical radiating elements along z-axis with static excitation I_n, $(n = 1, ..., N)$, and let each of them be energised periodically by the function

$$g_n(t) = \begin{cases} 1, & 0 \leq \tau_n^r < t < \tau_n^f \leq 1 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where τ_n^r and τ_n^f represent the starting and finishing normalized instants with respect to the modulating period T_p, respectively.

It is possible to demonstrate that the total amount sideband radiation can be computed by using a very effective closed-form equation, defined as

$$P_{SR} = 2\pi \sum_{n=1}^{N} |I_n|^2 [\tau_n (1 - \tau_n)] + 2\pi \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \Re \langle I_m I_n^* \rangle [\overline{\tau_{mn}} - \tau_n \tau_m] \operatorname{sinc} (k (z_m - z_n)) \tag{2}$$

where $\tau_n = \tau_n^f - \tau_n^r$ and $\overline{\tau_{mn}}$ denotes the overlapped duration between corresponding switch-on durations τ_n and τ_m.

1.2 Synthesis Technique

The synthesis technique is based on the a PSO-based strategy [1] aimed to define suitable rectangular-pulse waveforms exciting the elements composing the array and the distance between them, in order to reproduce a pattern with desired features in terms of SLL and BW and minimizing the amount of wasted power radiated in sideband. The cost function is defined as:

$$\Phi^{(k)}(\overline{\tau}, \overline{d}) = w_{SLL} \frac{\left|SLL^{(k)}(\overline{\tau}, \overline{d}) - SLL_{ref}\right|^2}{|SLL_{ref}|^2} + w_{BW} \frac{\left|BW^{(k)}(\overline{\tau}, \overline{d}) - BW_{ref}\right|^2}{|BW_{ref}|^2} + w_{SR} P_{SR}(\overline{\tau}, \overline{d})$$

where $SLL^{(k)}(\overline{\tau}, \overline{d})$ and SLL_{ref} are actual and target peak of sidelobe level, respectively, and $BW^{(k)}(\overline{\tau}, \overline{d})$ and BW_{ref} are actual and target $-3dB$ beamwidth, respectively, $\overline{\tau} = \{\tau_n;\ n = 1, ..., N\}$ is the set of pulse dutations and $\overline{d} = \{d_n = z_{n+1} - z_n;\ n = 1, ..., N - 1\}$ is the set of inter-element distance.

Numerical Results

2 TEST CASE 1 - $N_{ref} = 14$, $SLL_{ref} = -25dB$ (Dolph Comparison)

Goal

Time-modulated array synthesis through a *PSO*-based strategy optimizing the positions of the elements and the pulse durations exciting the elements of the array, jointly minimizing the sideband radiation. A comparison with a Dolph-Chebyshev pattern is proposed.

Reference Array Description

- Number of Elements: $N_{ref} = 14$
- Static Array Excitations: Uniform, $I_n = 1, \ \ n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$, $BW_{ref} = 8.51\,deg$
- Interelement Distance Range: $d_n = 0.5\lambda$

Optimized Array Description

- Number of Elements: $N_{opt} \in \{13, 12, 11, 10\}$
- Static Array Excitations: Uniform, $I_n = 1, \ \ n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: $SLL_{opt} = SLL_{ref} = -25\,dB$, $BW_{opt} = BW_{ref} = 8.51\,deg$
- Interelement Distance Range: $d_n \in [0.40 \div 1.0]\lambda$

Optimization Approach: PSO [1]

- Number of Variables: $X = 2N - 1$ (Normalized pulse duration τ_n, $n = 1, ..., N$, Interelement spacing d_n, $n = 1, ..., (N - 1)$)
- Number of Particles: $S = N$
- Number of Iterations: $M = 2000$
- Inertial Weight: $I_w = 0.4$
- Cost Function: SLL weight: $w_{SLL} = 10$, BW weight: $w_{BW} = 1$, SR weight: $w_{SR} = 1$

Reference Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$**,** $BW_{ref} = 8.51\,deg$ **-** $N_{opt} = 13$

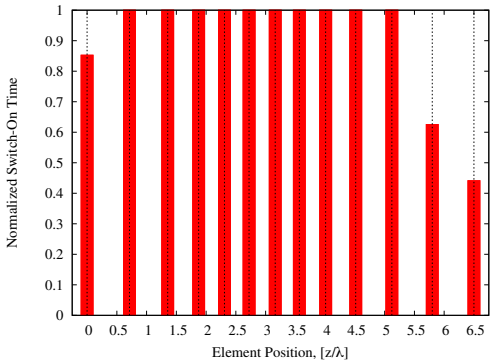

Fig. 1 - Pulse Sequence

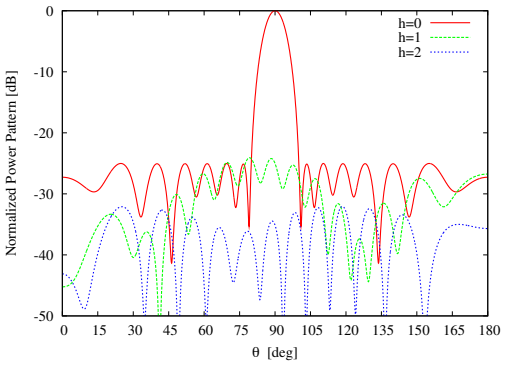

Fig. 2 - Beam Pattern

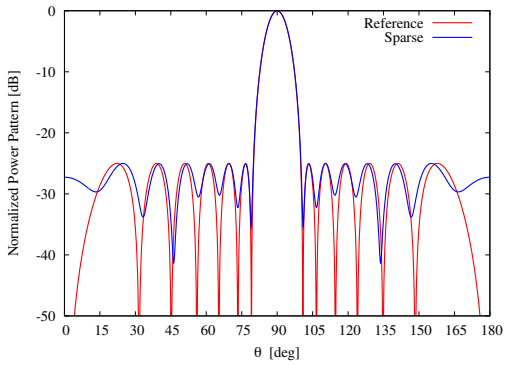

Fig. 3 - Beam Pattern comparison

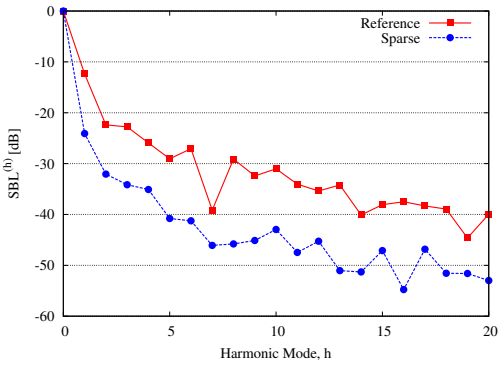

Fig. 4 - SBL

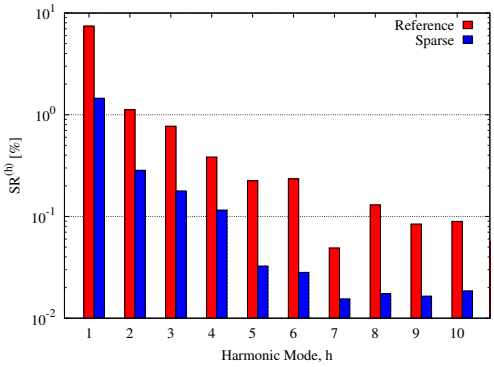

Fig. 5 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	14	6.50	-25.00	8.51	-12.36	22.22
$Sparse\,(PSO)$	13	6.50	-25.02	8.53	-24.09	4.54

Tab. I - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

Reference Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$**,** $BW_{ref} = 8.51\,deg$ **-** $N_{opt} = 12$

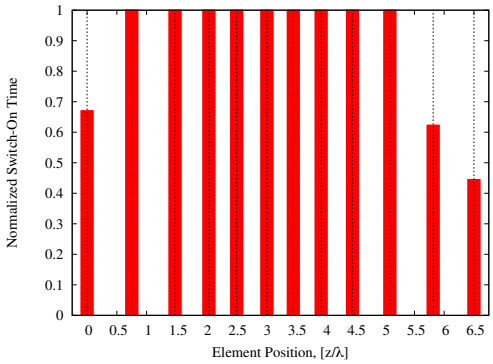

Fig. 6 - Pulse Sequence

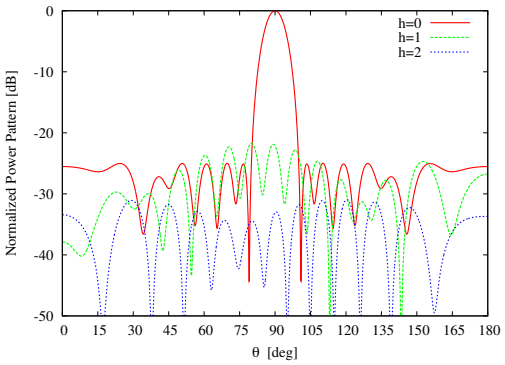

Fig. 7 - Beam Pattern

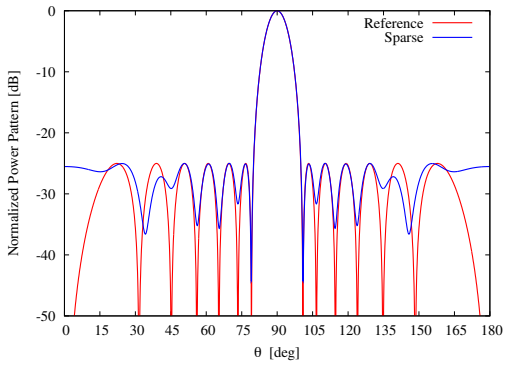

Fig. 8 - Beam Pattern comparison

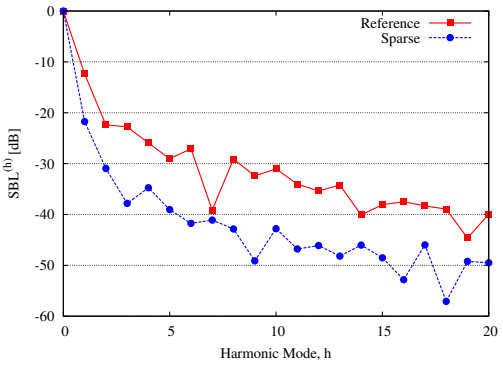

Fig. 9 - SBL

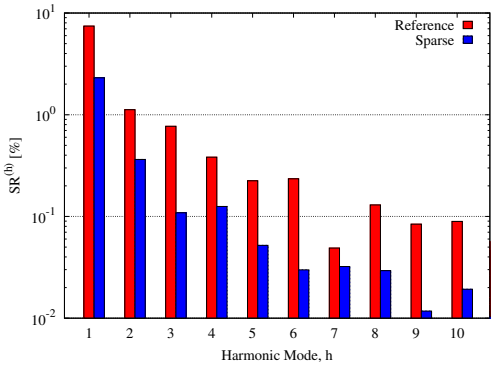

Fig. 10 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	14	6.50	-25.00	8.51	-12.36	22.22
$Sparse\,(PSO)$	12	6.50	-25.00	8.52	-21.73	6.43

Tab. II - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

Reference Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$**,** $BW_{ref} = 8.51\,deg$ **-** $N_{opt} = 11$

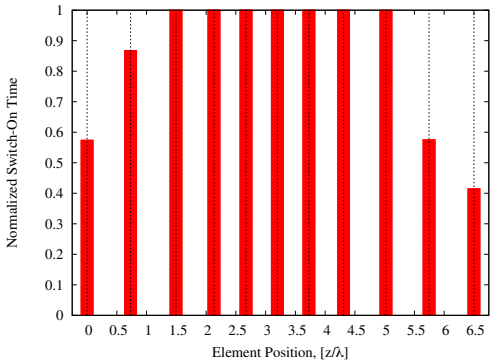

Fig. 11 - Pulse Sequence

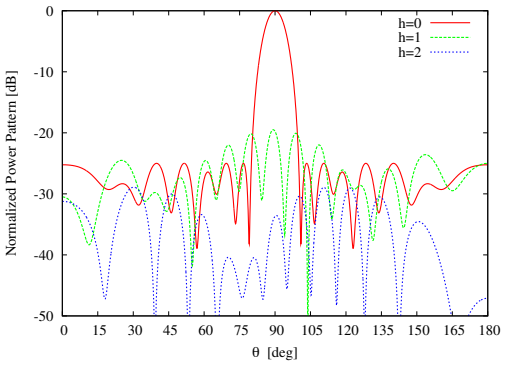

Fig. 12 - Beam Pattern

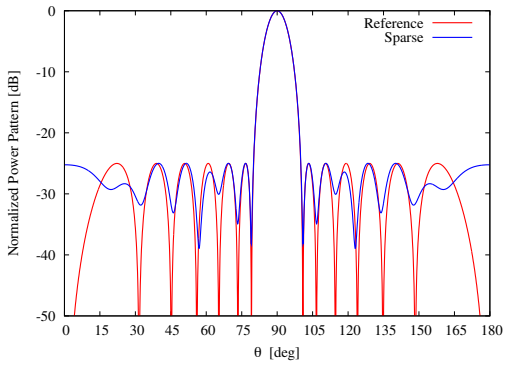

Fig. 13 - Beam Pattern comparison

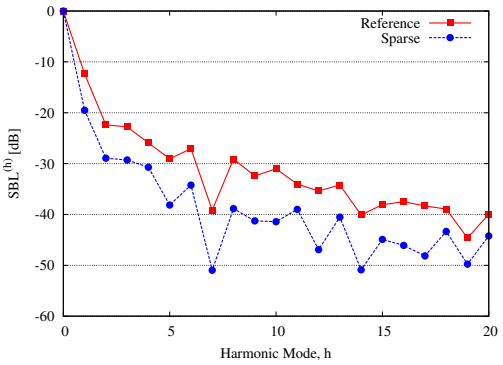

Fig. 14 - SBL

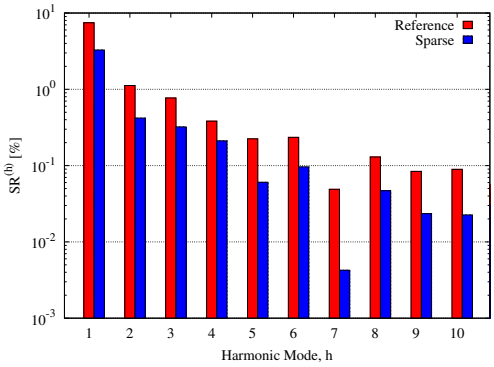

Fig. 15 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	14	6.50	-25.00	8.51	-12.36	22.22
$Sparse\,(PSO)$	11	6.50	-24.93	8.52	-19.52	9.41

Tab. III - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

Reference Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$**,** $BW_{ref} = 8.51\,deg$ **-** $N_{opt} = 10$

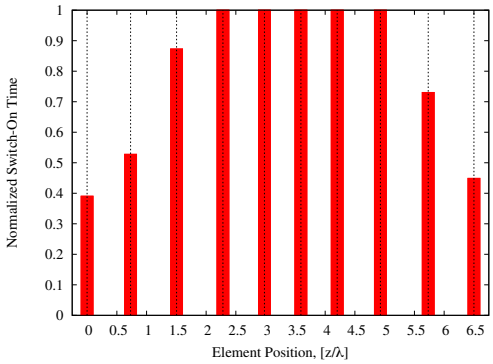

Fig. 16 - Pulse Sequence

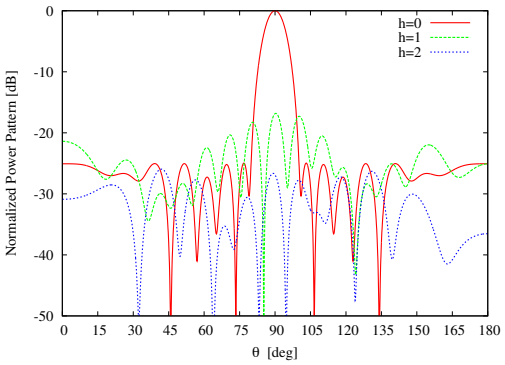

Fig. 17 - Beam Pattern

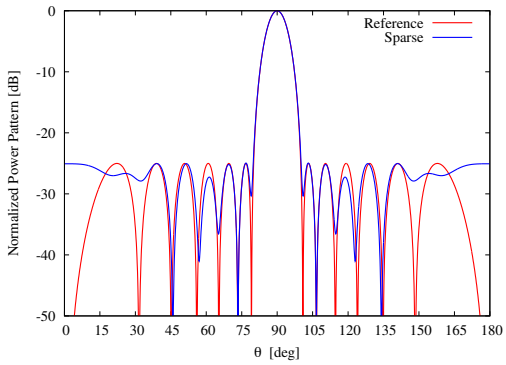

Fig. 18 - Beam Pattern comparison

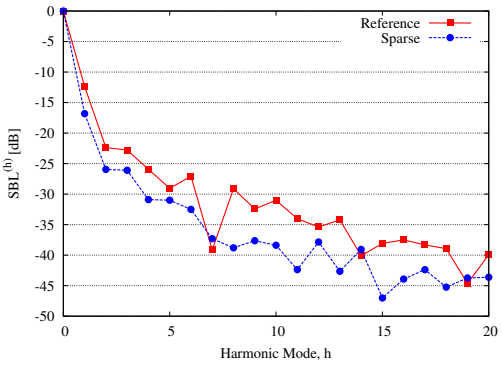

Fig. 19 - SBL

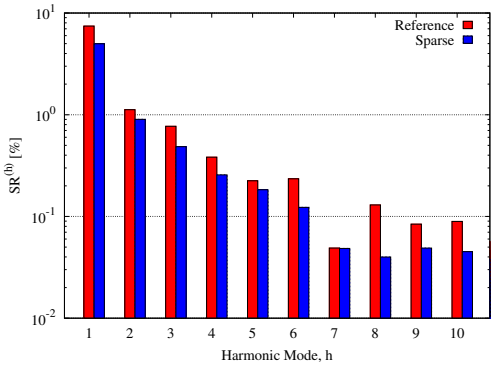

Fig. 20 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	14	6.50	-25.00	8.51	-12.36	22.22
$Sparse\,(PSO)$	10	6.50	-24.96	8.51	-16.81	14.96

Tab. IV - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

Reference Pattern: Dolph-Chebyshev, $SLL_{ref} = -25\,dB$, $BW_{ref} = 8.51\,deg$ - Resume

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	14	6.50	-25.00	8.51	-12.36	22.22
$Sparse\,(PSO)$	13	6.50	-25.02	8.53	-24.09	4.54
$Sparse\,(PSO)$	12	6.50	-25.00	8.52	-21.73	6.43
$Sparse\,(PSO)$	11	6.50	-24.93	8.52	-19.52	9.41
$Sparse\,(PSO)$	10	6.50	-24.96	8.51	-16.81	14.96

Tab. V - Optimized Solutions - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

3 TEST CASE 3 - $N_{ref} = 30$, $SLL_{ref} = -20dB$

3.1 Reducing the Number of Elements N_{opt}

Goal

Time-modulated array synthesis through a *PSO*-based strategy optimizing the positions of the elements and the pulse durations exciting the elements of the array, jointly minimizing the sideband radiation. A comparison with the results published in [1] is proposed.

Reference Array Description

- Number of Elements: $N_{ref} = 30$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: [Poli 2010], $SLL_{ref} = -20\,dB$, $BW_{ref} = 2.84\,deg$
- Interelement Distance Range: $d_n = 0.7\lambda$

Optimized Array Description

- Number of Elements: $N_{opt} = 22$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: $SLL_{opt} = SLL_{ref} = -20\,dB$, $BW_{opt} = BW_{ref} = 2.84\,deg$
- Interelement Distance Range: $d_n \in [0.40 \div 1.0]\lambda$

Optimization Approach: PSO [1]

- Number of Variables: $X = 2N - 1$ (Normalized pulse duration τ_n, $n = 1, ..., N$, Interelement spacing d_n, $n = 1, ..., (N - 1)$)
- Number of Particles: $S = N$
- Number of Iterations: $M = 2000$
- Inertial Weight: $I_w = 0.4$
- Cost Function: SLL weight: $w_{SLL} = 3$, BW weight: $w_{BW} = 3$, SR weight: $w_{SR} = 1$

Reference Pattern: [Poli 2010], $SLL_{ref} = -20\,dB$, $BW_{ref} = 2.84\,deg$ **-** $N_{opt} = 22$

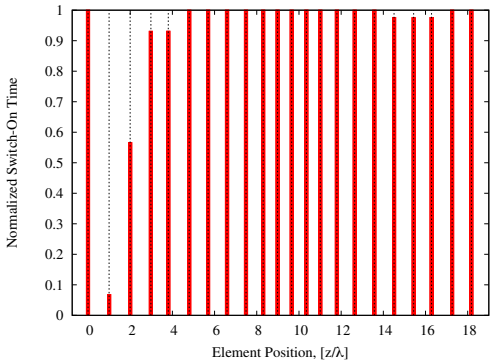

Fig. 21 - Pulse Sequence

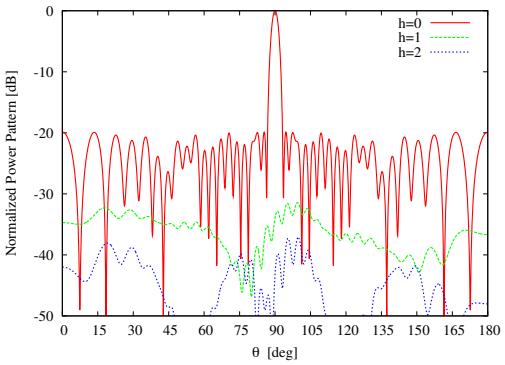

Fig. 22 - Beam Pattern

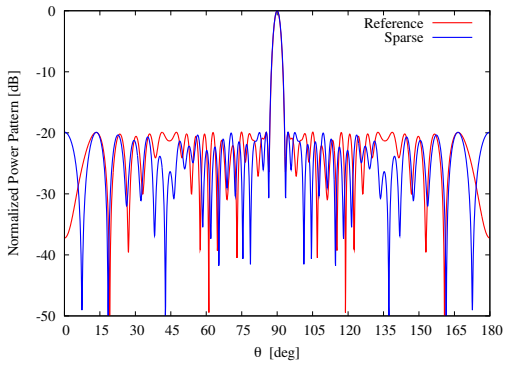

Fig. 23 - Beam Pattern comparison

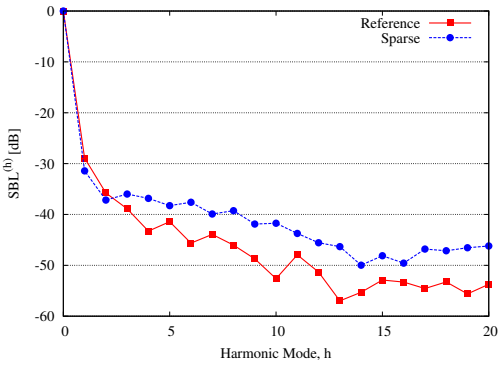

Fig. 24 - SBL

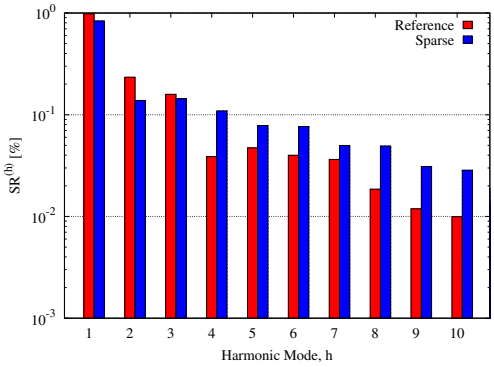

Fig. 25 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	30	20.30	-20.00	2.84	-28.91	3.57
$Sparse\,(PSO)$	22	18.17	-19.89	2.92	-31.47	3.54

Tab. VI - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

3.2 Reducing the Sideband Radiation SR

Goal

Time-modulated array synthesis through a *PSO*-based strategy optimizing the positions of the elements and the pulse durations exciting the elements of the array, jointly minimizing the sideband radiation. A comparison with the results published in [1] is proposed.

Reference Array Description

- Number of Elements: $N_{ref} = 30$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: [Poli 2010], $SLL_{ref} = -20\, dB$, $BW_{ref} = 2.84\, deg$
- Interelement Distance Range: $d_n = 0.7\lambda$

Optimized Array Description

- Number of Elements: $N_{opt} = 26$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: $SLL_{opt} = SLL_{ref} = -20\, dB$, $BW_{opt} = BW_{ref} = 2.84\, deg$
- Interelement Distance Range: $d_n \in [0.40 \div 1.0]\lambda$

Optimization Approach: PSO [1]

- Number of Variables: $X = 2N - 1$ (Normalized pulse duration τ_n, $n = 1, ..., N$, Interelement spacing d_n, $n = 1, ..., (N - 1)$)
- Number of Particles: $S = N$
- Number of Iterations: $M = 2000$
- Inertial Weight: $I_w = 0.4$
- Cost Function: SLL weight: $w_{SLL} = 3$, BW weight: $w_{BW} = 3$, SR weight: $w_{SR} = 1$

Reference Pattern: [Poli 2010], $SLL_{ref} = -20\,dB$**,** $BW_{ref} = 2.84\,deg$ **-** $N_{opt} = 26$

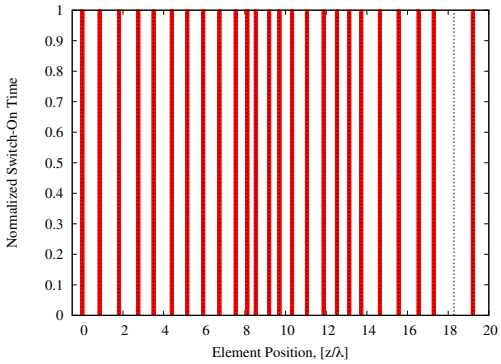

Fig. 26 - Pulse Sequence

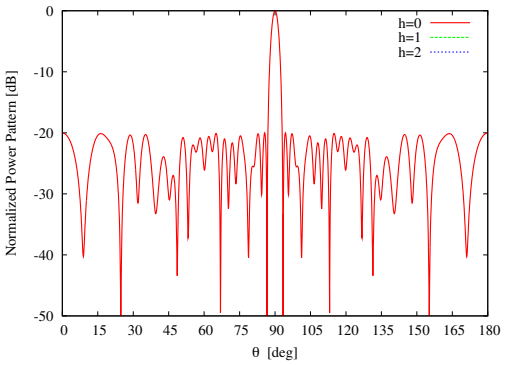

Fig. 27 - Beam Pattern

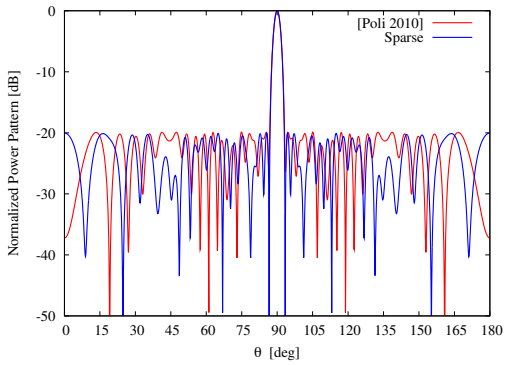

Fig. 28 - Beam Pattern comparison

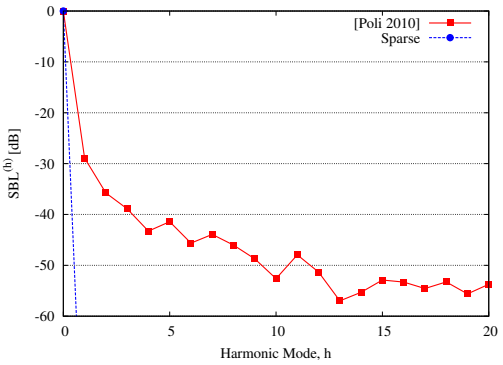

Fig. 29 - SBL

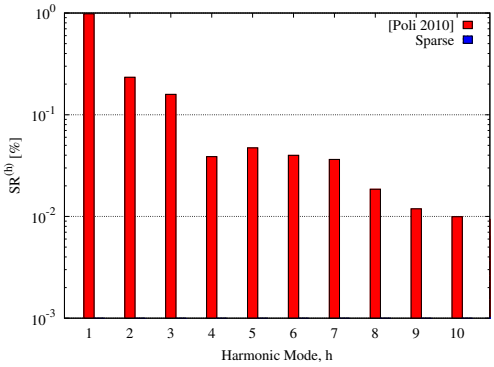

Fig. 30 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	30	20.30	-20.00	2.84	-28.91	3.57
$Sparse\,(PSO)$	26	19.22	-20.00	2.80	$-\infty$	0.00

Tab. VII - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

3.3 Reducing the Sidelobe Level SLL

Goal

Time-modulated array synthesis through a *PSO*-based strategy optimizing the positions of the elements and the pulse durations exciting the elements of the array, jointly minimizing the sideband radiation. A comparison with the results published in [1] is proposed.

Reference Array Description

- Number of Elements: $N_{ref} = 30$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: [Poli 2010], $SLL_{ref} = -20\, dB$, $BW_{ref} = 2.84\, deg$
- Interelement Distance Range: $d_n = 0.7\lambda$

Optimized Array Description

- Number of Elements: $N_{opt} = 22$
- Static Array Excitations: Uniform, $I_n = 1, \;\; n = 1, ..., N$
- Averaged Time-Modulated Target Pattern: $SLL_{opt} = SLL_{ref} = -20\, dB$, $BW_{opt} = BW_{ref} = 2.84\, deg$
- Interelement Distance Range: $d_n \in [0.40 \div 1.0]\lambda$

Optimization Approach: PSO [1]

- Number of Variables: $X = 2N - 1$ (Normalized pulse duration τ_n, $n = 1, ..., N$, Interelement spacing d_n, $n = 1, ..., (N - 1)$)
- Number of Particles: $S = N$
- Number of Iterations: $M = 2000$
- Inertial Weight: $I_w = 0.4$
- Cost Function: SLL weight: $w_{SLL} = 3$, BW weight: $w_{BW} = 3$, SR weight: $w_{SR} = 1$

Reference Pattern: [Poli 2010], $SLL_{ref} = -20\,dB$**,** $BW_{ref} = 2.84\,deg$ **-** $N_{opt} = 22$

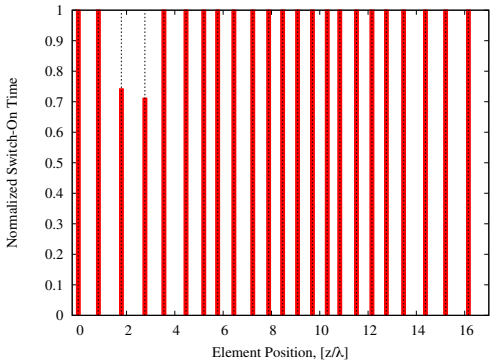

Fig. 31 - Pulse Sequence

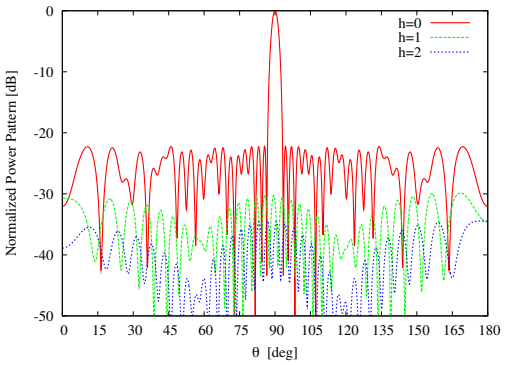

Fig. 32 - Beam Pattern

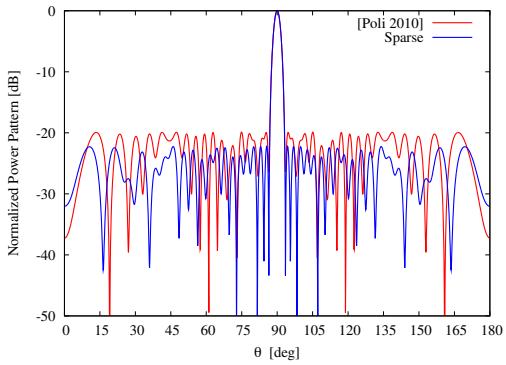

Fig. 33 - Beam Pattern comparison

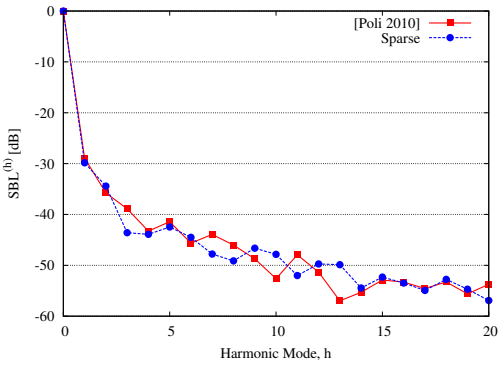

Fig. 34 - SBL

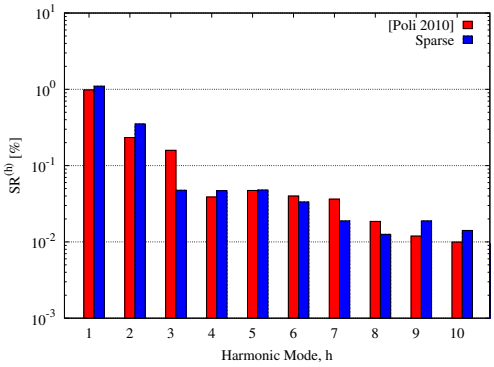

Fig. 35 - Sideband Radiation

	N	$L\,[\lambda]$	$SLL\,[dB]$	$BW\,[deg]$	$SBL\,[dB]$	$SR\,[\%]$
$Reference$	30	20.30	-20.00	2.84	-28.91	3.57
$Sparse\,(PSO)$	26	18.91	-22.21	2.85	-29.84	3.60

Tab. VIII - Optimized Solution - Features: Number of Elements (N), Array Aperture (L), Sidelobe Level (SLL), -3dB Beamwidth (BW), Sideband Level (SBL) and Sideband Radiation (SR)

References

[1] L. Poli, P. Rocca, L. Manica, and A. Massa, "Handling sideband radiations in time-modulated arrays through particle swarm optimization," *IEEE Trans. Antennas Propag.*, vol. 58, no. 4, pp. 1408-1411, Apr. 2010.

[2] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "A multi-stage approach for the synthesis of sub-arrayed time modulated linear arrays," IEEE Trans. Antennas Propag., vol. 59, no. 9, pp. 3246-3254, Sep. 2011.

[3] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Harmonic beamforming in time-modulated linear arrays," IEEE Trans. Antennas Propag., vol. 59, no. 7, pp. 2538-2545, Jul. 2011.

[4] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "Adaptive nulling in time-varying scenarios through time-modulated linear arrays," IEEE Antennas Wireless Propag. Lett., vol. 11, pp. 101-104, 2012.

[5] P. Rocca, L. Poli, and A. Massa, "Instantaneous directivity optimization in time-modulated array receivers," IET Microwaves, Antennas & Propagation, vol. 6, no. 14, pp. 1590-1597, Nov. 2012.

[6] L. Poli, P. Rocca, and A. Massa, "Sideband radiation reduction exploiting pattern multiplication in directive time-modulated linear arrays," IET Microwaves, Antennas & Propagation, vol. 6, no. 2, pp. 214-222, 2012.

[7] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Adaptive nulling in time-modulated linear arrays with minimum power losses," IET Microwaves, Antennas & Propagation, vol. 5, no. 2, pp. 157-166, 2011.

[8] L. Poli, P. Rocca, L. Manica, and A. Massa, "Time modulated planar arrays - Analysis and optimization of the sideband radiations," IET Microwaves, Antennas & Propagation, vol. 4, no. 9, pp. 1165-1171, 2010.

[9] P. Rocca, L. Poli, G. Oliveri, and A. Massa, "Synthesis of time-modulated planar arrays with controlled harmonic radiations," Journal of Electromagnetic Waves and Applications, vol. 24, no. 5/6, pp. 827-838, 2010.

[10] L. Manica, P. Rocca, L. Poli, and A. Massa, "Almost time-independent performance in time-modulated linear arrays," IEEE Antennas Wireless Propag. Lett., vol. 8, pp. 843-846, 2009.

[11] L. Poli, P. Rocca, G. Oliveri, and A. Massa, "Failure correction in time-modulated linear arrays," IET Radar, Sonar & Navigation, vol. 8, no. 3, pp. 195-201, Mar. 2014.

[12] P. Rocca, Q. Zhu, E. T. Bekele, S. Yang, and A. Massa, "4D arrays as enabling technology for cognitive radio systems," IEEE Transactions on Antennas and Propagation - Special Issue on "Antenna Systems and Propagation for Cognitive Radio," vol. 62, no. 3, pp. 1102-1116, Mar. 2014.