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A Reconstruction Procedure for Microwave Nondestructive Evaluation based on a Numerically Computed Green's Function

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Abstract* – This paper describes a new microwave diagnostic tool for nondestructive evaluation. The approach, developed in the spatial domain, is based on the numerical computation of the inhomogeneous Green's function in order to fully exploit all the available a-priori information of the domain under test. The heavy reduction of the computational complexity of the proposed procedure (with respect to standard procedures based on the free-space Green's function) is also achieved by means of a customized hybrid-coded genetic algorithm. In order to assess the effectiveness of the method, the results of several simulations are presented and discussed.

Keywords: *Imaging Systems, Imaging Processing, Material Characterization, Green's Function, Genetic Algorithms*

I. INTRODUCTION

In many tomographic approaches, in which electromagnetic waves are used to inspect dielectrics, the inversion procedures are developed in the spatial domain (as opposed to spectral domain). In practical applications, the main interest is usually represented by the inspection of inhomogeneous scatterers of arbitrary (bounded) cross sections (e.g., in microwave nondestructive testing and evaluation (NDT/NDE) [1]-[7].) In very few cases, the objects under test are weak enough to allow the practical use of simplified [8][9] or closed-form solutions [10].

Recently, the development of reconstruction procedures for microwave tomography has been addressed by resorting to the numerical discretization of the integral equations of the inverse scattering problem. The Fredholm equation of the first kind (i.e., the data equation governing the relation among the scattering potential, the total electric field inside the body, and the scattered electric field at the receivers) results, as it is well known, in a highly nonlinear and ill-posed inverse problem. The discretized version of this equation is affected by a severe ill-conditioning. The problem solution is generally addressed by associating to the data equation the so-called state equation, i.e. the equation relating the incident and total fields inside the scatterer. A suitable functional is constructed (often arbitrarily), whose minimization correspond to the attempt of fulfilling as much as possible the state and data equations. To this end, present authors proposed in [11]-[13] the application of global optimization approaches, and, in particular, the use of a hybrid-coded genetic algorithm (GA) [11]. Due to its flexibility, a GA is able to deal with integer as well as real variables at the same time and does not require neither differentiability nor continuity of the cost function to be minimized. However, as it well known, the main drawback of such very appealing methods (simple to be implemented, robust, and insensitive to details of the cost function) is the high computational load. Although the continuous increasing in the computational power of new computers tends to alleviate this problem, a pixel representation of the cross section of an unknown complex object is still a difficult task.

However, the GA presents other advantages over several deterministic techniques. It allows the simple and straightforward insertion of *a-priori* information into the model. The exploitation of *a-priori* information is very important in practical applications allowing a reduction of the search space sampled by the optimization procedure and, consequently, an increase of the convergence rate of the iterative process.

An example is represented by the NDE problem considered in the present paper. In this case, the object to be detected is only a defect in an otherwise known object. Consequently, the inverse scattering problem is notably simplified and the use of a GA for the retrieval of some characteristic parameters of the defect (position, dimensions, orientation, etc.) is convenient. In this framework, the main novelty of the proposed approach lies in the use of the Green's function for the unperturbed geometry, which can be numerically computed off-line and once for all. As a result, only space region occupied by the defect is the "investigation area" considered during the minimization process and the chromosomes of the GA (coding the problem unknowns) result greatly shortened allowing a significant reduction of the computational burden.

The paper is organized as follows: In Section II, the mathematical formulation of the proposed approach is presented. Section III gives a description of the optimization procedure based on a customized genetic algorithm pointing out the key-points of its application to the NDE framework. Finally, in Section IV, selected numerical results, concerning both noiseless and noisy environments as well as lossy and lossless investigation domains, are reported in order to show the capabilities and current limitations of the method in providing accurate defect localizations and reconstructions.

II. MATHEMATICAL FORMULATION

Let us consider an investigation area S modeled by a *scattering potential,* γ, given by

$$\gamma(\mathbf{r}) = (j\omega\mu_0)^{-1}\{[\sigma(\mathbf{r}) - \sigma_e] + j\omega\varepsilon_e[\varepsilon(\mathbf{r}) - 1]\} \tag{1}$$

where (σ, ε) and $(\sigma_e, \varepsilon_e)$ are the conductivities and relative dielectric permittivities inside and outside *S*, respectively. The region *S* is illuminated by a set of transverse magnetic (TM) incident fields, $\mathbf{E}_i^{inc}(\mathbf{r}), i = 1,...,I$. The scattered data are collected in *M* measurement points (arranged around the object under test), $\mathbf{E}_{ij}^{scat}(\mathbf{r}_j), i = 1,...,I, j = 1,...,M$. The inverse problem can be recast as an optimization problem, where a functional is to be minimized [11]:

$$\begin{aligned}\Im\{\gamma(\mathbf{r}), E_i^{tot}(\mathbf{r})\} = \Im_{Data}\{\gamma(\mathbf{r}), E_i^{tot}(\mathbf{r})\} \\ + \Im_{State}\{\gamma(\mathbf{r}), E_i^{tot}(\mathbf{r})\}\end{aligned} \tag{2}$$

being

$$\Im_{Data}\left\{\gamma(\mathbf{r}), E_i^{tot}(\mathbf{r})\right\} = \frac{\left\| \mathcal{L}_2\left\{\gamma(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\} - \mathbf{E}_{ij}^{scat}\left(\mathbf{r}_j\right)\right\|^2}{\left\|\mathbf{E}_{ij}^{scat}\left(\mathbf{r}_j\right)\right\|^2}$$

and

$$\Im_{Data}\left\{\gamma(\mathbf{r}), E_i^{tot}(\mathbf{r})\right\} = \frac{\left\| \mathcal{L}_1\left\{\gamma(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\} + \mathbf{E}_i^{inc}(\mathbf{r})\right\|^2}{\left\|\mathbf{E}_i^{inc}(\mathbf{r})\right\|^2}$$

where $\mathcal{L}_1\left\{\gamma(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\}$ and $\mathcal{L}_2\left\{\gamma(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\}$ are nonlinear operators whose unknown functions are $\gamma = \gamma\{\varepsilon(\mathbf{r}), \sigma(\mathbf{r})\}$ (which, in NDE applications, contains the information on the unknown defect) and $\mathbf{E}_i^{tot}(\mathbf{r}), i = 1,..., I$. These operators are defined in details in [11]-[13] for the case in which the kernel is the free space Green's function [14]. In this paper, a numerically computed Green's function for the unperturbed configuration (the configuration without the defect) is considered. The Green's function satisfies the following equation:

$$\Gamma\left(\mathbf{r}/\mathbf{r}'\right) = \Gamma_0\left(\mathbf{r}/\mathbf{r}'\right) + \iint_S \gamma_I(\mathbf{x})\Gamma\left(\mathbf{x}/\mathbf{r}'\right)\Gamma_0\left(\mathbf{r}/\mathbf{x}\right)d\mathbf{x} \quad (3)$$

where $\Gamma\left(\mathbf{r}/\mathbf{r}'\right)$ is the inhomogeneous Green's function, $\Gamma_0\left(\mathbf{r}/\mathbf{r}'\right)$ is the free space Green's function, and $\gamma_I(\mathbf{r}) = \left(j\omega\mu_0\right)^{-1}\left\{\left[\sigma_I(\mathbf{r}) - \sigma_e\right] + j\omega\varepsilon_e\left[\varepsilon_I(\mathbf{r}) - 1\right]\right\}$ is the scattering potential of the unperturbed geometry. Equation (3) can be solved off-line and once for all by means of the moment method. As an example of these computations, Figure 1 shows the amplitudes of the inhomogeneous Green's function for a point source located at the center of the investigation domain and in correspondence with different host medium configurations (two dimensional case).

After discretization of the continuous model, $\Im\left\{\gamma(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\}$ (equation (2)) is minimized by means of a suitable GA-based procedure [15] able to efficiently exploit the features of the proposed approach.

III. GA-BASED PROCEDURE

Thanks to the numerical knowledge of the Green's function for the unperturbed scenario, let us model the defect by means of a differential scattering potential, $\tilde{\gamma}$, defined as $\tilde{\gamma}(\mathbf{r}) = \gamma(\mathbf{r}) - \gamma_I(\mathbf{r})$, which completely describes the dielectric profile of the investigation domain and whose support is limited to the crack area (on the contrary, in [11] the whole dielectric configuration of the investigation domain is unknown). Then, by defining a suitable parameterization of the defect shape, the set of unknown crack parameters can be suitably represented by means of a small subset of discrete variables $\tilde{\gamma}(\mathbf{r}) \Rightarrow (\ell_j, j = 1,..., J)$, being ℓ_j the j-*th* defect discrete descriptor. Consequently, the arising unknown array results in a variable-length hybrid-encoded "individual" obtained by concatenating the code of discrete and real-valued parameters [11]:

$$\left\{\tilde{\gamma}(\mathbf{r}), \mathbf{E}_i^{tot}(\mathbf{r})\right\} \Rightarrow \left\{\ell_j, j = 1, \ldots, J; \mathbf{E}_i^{tot}(\mathbf{r})\right\} \quad (4)$$

In order to obey the mathematical properties of the array unknown representation (relation (4)), suitable GA operators are considered. Binary tournament selection [16] and real/binary double point crossover [15][17] are used for the selection and the crossover, respectively. The mutation is performed with probability P_m on an individual and consists in perturbing one element of its genetic sequence. If the element is a discrete variable, it is changed randomly in a limited set of values. Otherwise, the mutation rule proposed in [11] is adopted.

IV. NUMERICAL VALIDATION

In order to assess the effectiveness of the proposed approach (in the following indicated by *IGA*) pointing out its efficiency by a computational point of view, some numerical simulations have been performed. In this Section, selected numerical results are presented in order to demonstrate the two main features of the approach:

(*a*) the accuracy in the crack detection and estimation;

(*b*) the reduction of the computational load with respect to the use of the method (namely the *FGA*) presented in [11].

As far as the test case is concerned, a two-dimensional scenario is taken into account where a void crack lies in a square (side: $0.8\lambda_0$) homogeneous host medium. Figure 2 shows the reconstruction of a square crack $0.2\lambda_0$ -sided inside a lossy host medium ($\sigma_I = 0.25\ [S/m]$, $\varepsilon_I = 2.0$) during the iterative reconstruction process. It can be observed that the location of the center of the defect is accurate in both cases (*IGA* and *FGA*). As far as the area estimation is concerned, slightly differences in the final reconstruction (i.e., at the convergence iteration $k = K^*$) occur. For completeness, Figures 2(*a*) and 2(*e*) show the original and the best initial trial configurations, respectively.

The imaging capabilities of the two approaches are rather different for higher values of the conductivity of the host medium. Figure 3 shows the images of the reconstructed distributions at different iterations in the case in which $\sigma_I = 0.5\ [S/m]$. The position of the defect is accurately estimated and its area slightly under-estimated when the *IGA* is used (Fig. 3(*l*)). On the contrary, the final result reached by the *FGA* (Fig. 3(*e*)) is very poor for the location as well as for the shape reconstruction, even after a large number of iterations ($K^*_{FGA} = 200$ and $K^*_{IGA} = 165$). Consequently, it can be preliminarily inferred that the *IGA* procedure produces better results than the *FGA* approach. This statement is clearly confirmed and generalized by the results obtained by performing an exhaustive set of numerical simulations varying the environment conditions (the values of the signal-to-noise ratio in the range between 2.5 *dB* and 50 *dB*) and the dielectric characteristics of the host medium (in particular, its conductivity between 0.1 [S/m] and 1.0 [S/m]). Due to the stochastic nature of the optimization algorithm, a set of 10 simulations has been carried out for each scenario under test.

In order to quantitatively assess the effectiveness of the IGA-method in comparison with the FGA-approach, two error figures have been defined:

$$\delta_c = \frac{\sqrt{(x-\hat{x})^2 - (y-\hat{y})^2}}{d_{\max}} \times 100 \quad (5)$$

$$\delta_a = \left| \frac{A - \hat{A}}{A} \right| \times 100 \tag{6}$$

where (x, y) and $(\hat{x}, \hat{y})$ are actual and estimated coordinates of the center of the crack, respectively; $d_{\max}$ being the maximum error in defining the crack center; A and $\hat{A}$ are the estimated and actual crack areas, respectively.

Figure 4 shows a three-dimensional color-level representation of δ_c and δ_a. As preliminary drawn from Figs. 2 and 3, the IGA approach generally outperforms the FGA method especially in the estimation of the crack area and in correspondence with a more realistic industrial environment (where small values of the SNR generally arise). In particular, it can be observed that $\{\delta_a\}_{IGA} \leq 50$ when in general $\{\delta_a\}_{FGA} > 50$.

Finally, in order to quantify the computational effectiveness of the proposed approach, the following parameter is evaluated:

$$\Delta_{conv} = \frac{K^*_{FGA} - K^*_{IGA}}{K^*_{FGA}} \times 100 \tag{7}$$

and the obtained results are reported in Figure 5. The plot clearly indicates that the convergence rate of the *IGA* is generally greater than that of the *FGA*. As expected, the differences increase in correspondence with lower SNR values and for lossy host regions.

For completeness, in order to give an idea of the time saving allowed by the *IGA* method, Table I gives the statistics of the time required for each iteration of the optimization procedure. For comparison purposes, also the values for the FGA approach are reported, too. As can be observed, it results that, on an average, an *IGA* iteration took approximately $2/5$ of the time necessary for the *FGA* iterative step.

V. CONCLUSIONS AND FUTURE DEVELOPMENTS

An innovative approach for the crack detection in known host medium has been presented. In order to fully exploit the knowledge of the scenario under test, a new formulation based on the numerical computation of the Green's function for the unperturbed configuration has been proposed. Moreover, the new formulation requires the definition of a customized minimization procedure based on a genetic algorithm which results in an heavy computational saving. This fact, confirmed by several numerical simulations, clearly indicates a possibility for the quasi real-time implementation of the proposed technique in real-word monitoring of industrial processes. To this end, further improvements and generalizations are mandatory. Let us consider the extension to more general crack shapes (and consequently the need for more complete and complicated crack parameterizations), or the possibility of dealing with multiple defects in the same host medium, or (in some specific industrial applications) the increase of the resolution capabilities. In this framework, the authors are currently involved in developing different software tools and experimental apparatus.

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FIGURE CAPTIONS

Figure 1. Computed inhomogeneous Green's function for a point source located at the center of the host medium characterized by: (*a*) $\sigma_I = 0.0$ $[S/m]$, $\varepsilon_I = 1.0$ (free space Green's function), (*b*) $\sigma_I = 0.0$ $[S/m]$, $\varepsilon_I = 2.0$, (*c*) $\sigma_I = 0.25$ $[S/m]$, $\varepsilon_I = 2.0$, and (*d*) $\sigma_I = 0.50$ $[S/m]$, $\varepsilon_I = 2.0$.

Figure 2. Reconstruction of a void crack in a lossy host medium ($\sigma_I = 0.25$ $[S/m]$).

Figure 3. Reconstruction of a void crack in a lossy host medium ($\sigma_I = 0.50$ $[S/m]$).

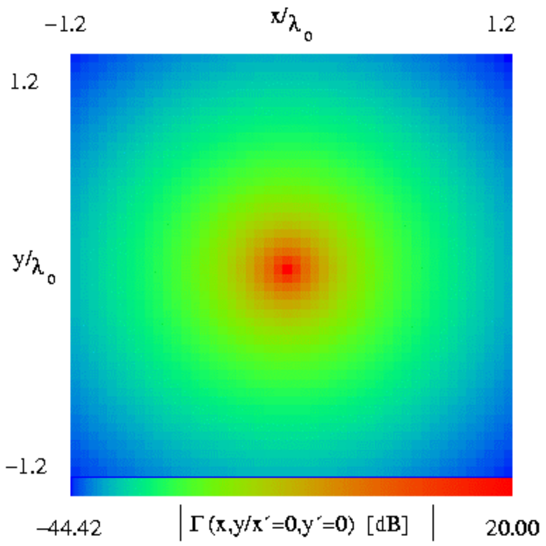
Figure 4. Errors in the crack reconstruction for different SNR values and for different conductivities of the host medium (σ_I): (*a*) δ_c and (*b*) δ_a.

Figure 5. Reconstruction of a void crack in a lossy host medium (noisy environment) – Converge rate estimation.

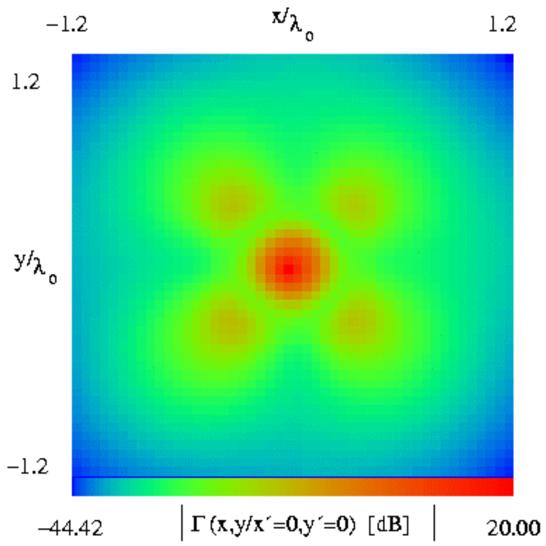
TABLE CAPTIONS

Table I. Statistics of the time required for each iteration of the minimization procedure.

FIGURES



(*a*)



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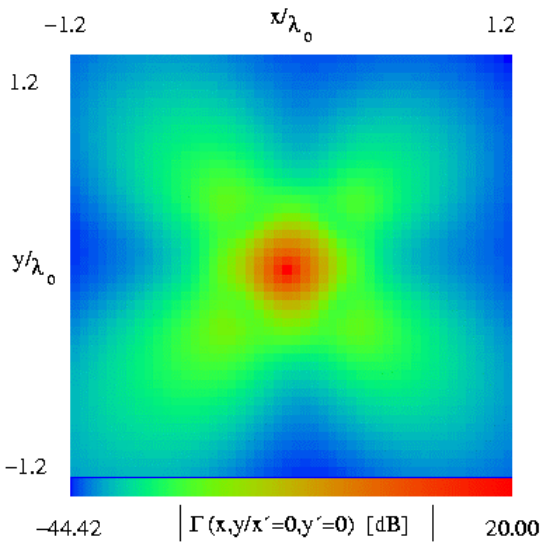

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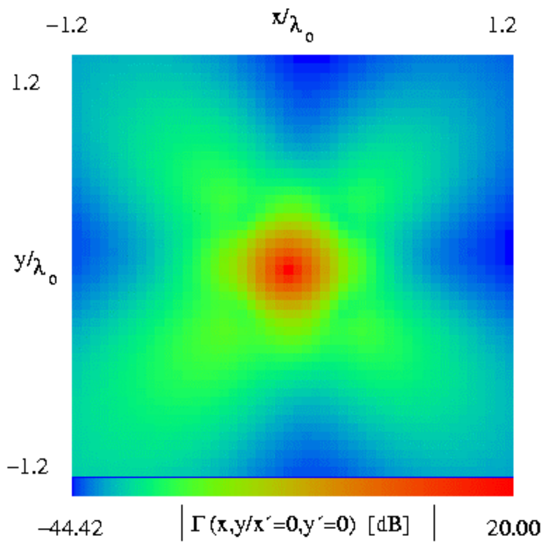
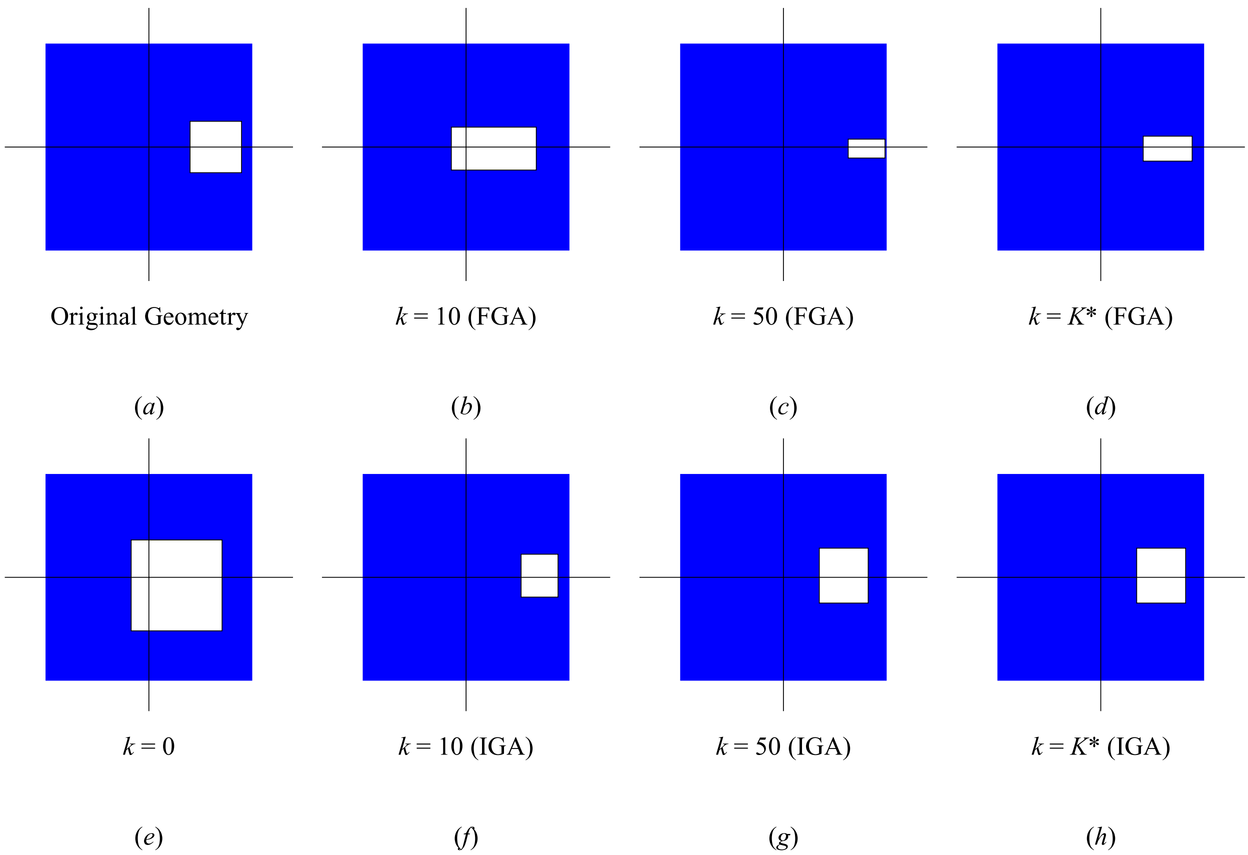


(*d*)

Fig. 1 - S. Caorsi *et al.*, "A reconstruction procedure for microwave ..."



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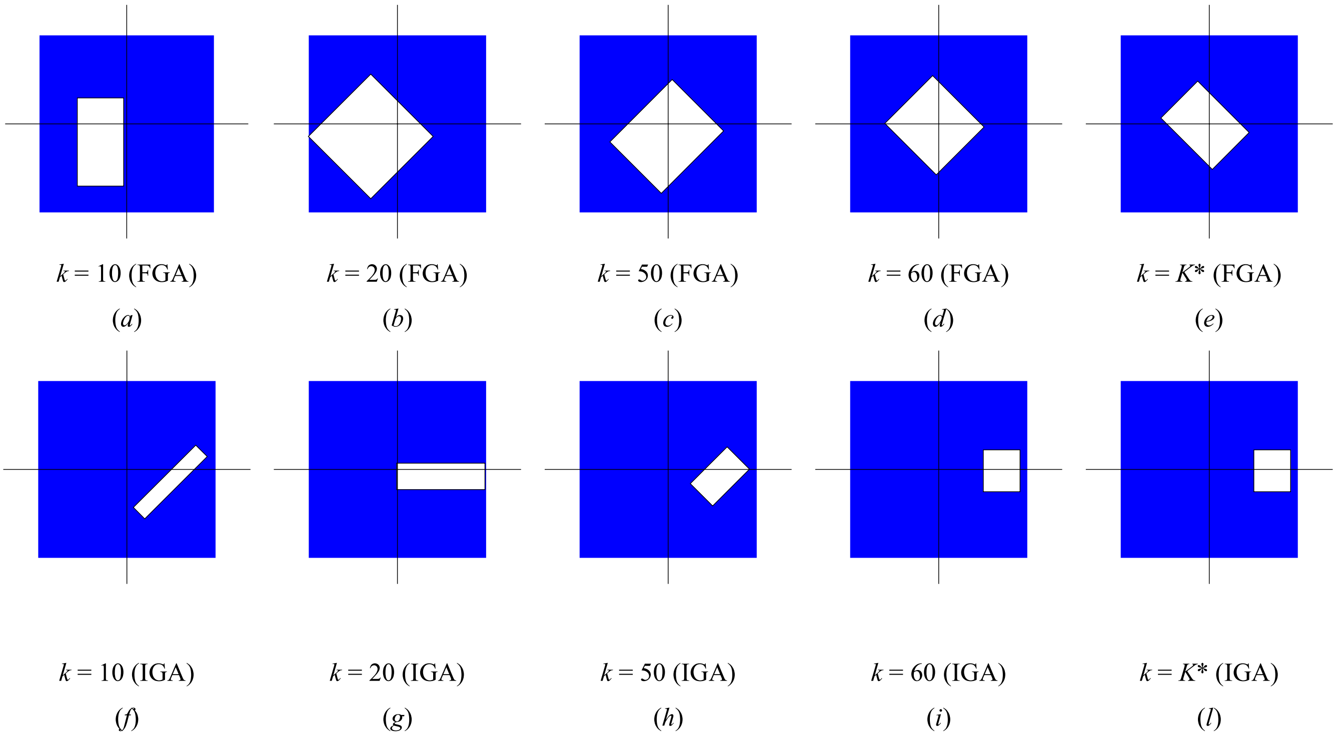

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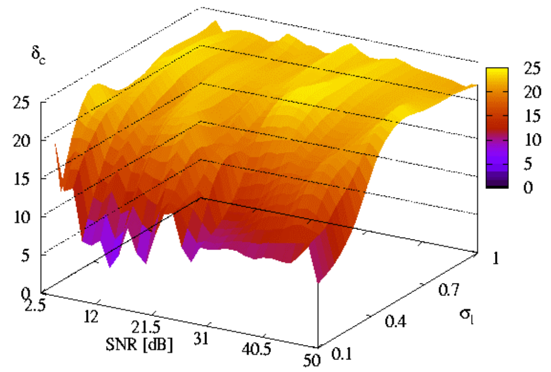
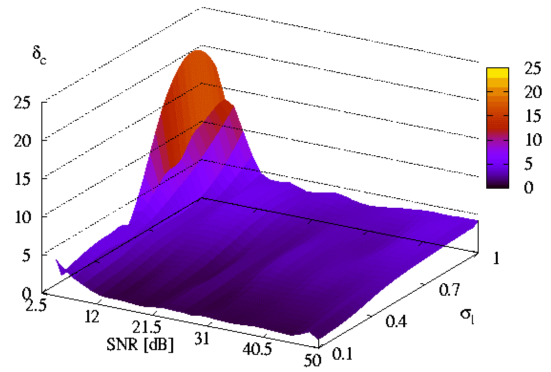


(*a*)



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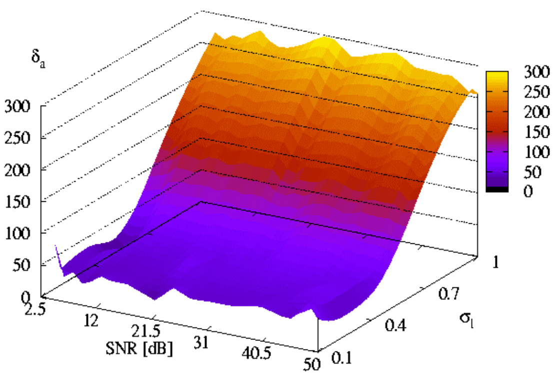

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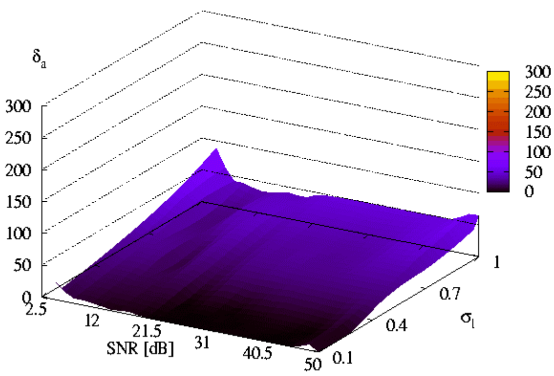

(*d*)

Fig. 4 - S. Caorsi *et al.*, "A reconstruction procedure for microwave ..."

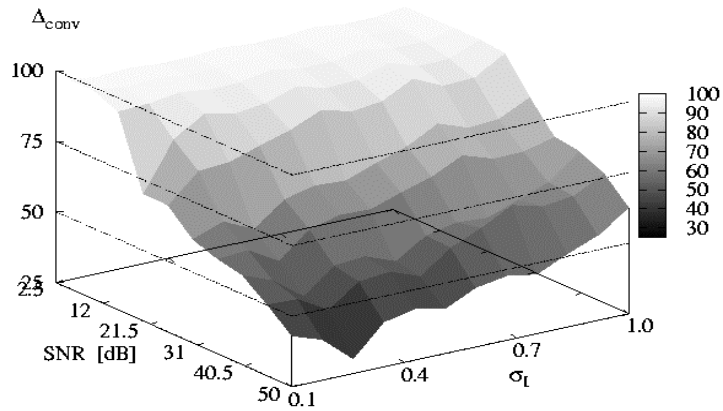

Fig. 5 - S. Caorsi *et al.*, "A reconstruction procedure for microwave ..."

	FGA	*IGA*
Minimum Value	2.0 [sec]	0.4 [sec]
Average Value	3.36 [sec]	1.35 [sec]
Maximum Value	3.88 [sec]	1.70 [sec]

Tab. I - S. Caorsi *et al.*, "A reconstruction procedure for microwave ..."