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ANT COLONY BASED HYBRID APPROACH FOR OPTIMAL COMPROMISE SUM-DIFFERENCE PATTERNS SYNTHESIS

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Abstract

Dealing with the synthesis of monopulse array antennas, many stochastic optimization algorithms have been used for the solution of the so-called optimal compromise problem between sum and difference patterns when sub-arrayed feed networks are considered. More recently, hybrid approaches, exploiting the convexity of the functional with respect to a sub-set of the unknowns (i.e., the sub-array excitation coefficients) have demonstrated their effectiveness. In this letter, an hybrid approach based on the Ant Colony Optimization (ACO) is proposed. At the first step, the ACO is used to define the sub-array membership of the array elements, while, at the second step, the sub-array weights are computed by solving a convex programming problem.

1 Introduction

In the framework of multi-beam antenna synthesis, several strategies have been proposed for the optimal design of monopulse arrays able to simultaneously generate a sum and multiple difference patterns [1][2]. Because of the complexity of the feed network when using independent sum and difference excitations, compromise approaches have been investigated to identify suitable trade-off solutions that guarantee reduced manufacturing costs and satisfactory radiation performances.

The feeding of the array elements by means of sub-arrays with a reduced number of control elements has been widely adopted [3]. As regards to compromise synthesis [4], the sum channel is fed by a complete and independent module to obtain an optimal pattern, while the sub-arrays outputs are properly weighted and combined to synthesize sub-optimal difference beams.

Since the functional describing the "optimal sum/difference compromise" problem is characterized by the presence of many local minima, global optimization approaches have been used (e.g., simulated annealing [5], genetic algorithms [6], and differential evolution [7][8]). By exploiting the convexity of the functional with respect to the sub-array weights [9], an effective hybrid approach has been presented in [10] where the synthesis problem is recast as a convex programming optimization once the sub-array configuration is given. Although the set of "optimal" sub-array excitations can be unequivocally determined for a fixed element grouping, the solution of the aggregation problem still remains non-unique and stochastic optimization approaches cannot be profitably used without a non-negligible computational burden because of the wide number of admissible sub-array configurations.

To overcome this drawback, the knowledge of the optimal sum and difference excitations has been exploited in [11] by addressing the compromise synthesis through an excitation matching procedure. In such a way, the dimension of the solution space has been greatly reduced and an efficient use of global optimization algorithms enabled also thanks to a suitable graph-based representation of the whole set of admissible sub-array configurations. In such a framework, a strategy based on Ant Colony Optimization (ACO) [12] has been preliminary presented in [13] to effectively sample the solution space. Successively, the approach has been extensively validated on a large set of numerical examples [14] pointing out a loss of the control of the behavior of the sidelobes without proper countermeasures.

Similarly to [10][15][16], this letter describes a hybrid two-step procedure aimed at synthesizing an optimal compromise solution in terms of both reference matching and sidelobe control. Unlike [15][16], the grouping of the array elements is determined by solving the excitation matching problem through the ACO-based global optimization [14]. At the second step, the computation of the sub-array weights is recast as the solution of a standard quadratic programming aimed at enforcing peak sidelobe level control.

The letter is organized as follows. In Sect. 2, the compromise problem is mathematically formulated and the hybrid synthesis procedure is described, as well. To demonstrate the effectiveness of the proposed approach, some representative results from an extensive set of experiments are reported and discussed in Sect. 3. Eventually, some conclusions are drawn in Sect. 4.

2 Mathematical Formulation

Let us consider a uniform linear array of $N = 2 \times M$, $m = 1, ..., M$, equally-spaced radiating elements. The element excitations generating the sum and the difference patterns are supposed to be real and symmetric with respect to the center of the antenna to steer both beams at broadside.

Dealing with isotropic sources and considering the compromise solution with an optimal sum and a sub-optimal difference, the sum mode radiated by the array is given by [10]

$$AF^s \left(\theta \right) = 2 \sum_{m=1}^{M} \alpha_m cos \left[\frac{2m - 1}{2} kdsin \left(\theta \right) \right], \tag{1}$$

while the difference mode turns out to be

$$AF^d \left(\theta \right) = j2 \sum_{m=1}^{M} b_m sin \left[\frac{2m - 1}{2} kdsin \left(\theta \right) \right] \tag{2}$$

where $\alpha_m = \alpha_{-m}$ and $b_{-m} = -b_m$, $m = 1, ..., M$, are the set of optimal sum and compromise difference excitations [4], respectively. Moreover, $k = \frac{2\pi}{\lambda}$, λ and d being the free space wavelength and the array inter-element distance, respectively, and θ is the angle with respect to boresight.

Following the guidelines of the excitation matching strategy described in [11], the compromise problem is recast as the minimization of the following cost function

$$\Psi \left(\underline{C}, \underline{W} \right) = \frac{1}{M} \sum_{m=1}^{M} \alpha_m^2 \left[\frac{\beta_m}{\alpha_m} - \sum_{q=1}^{Q} \delta_{c_m q} w_q \right]^2 \tag{3}$$

where the unknown quantities $\underline{C} = \{c_m \in [1, Q]; \, m = 1, \ldots, M\}$ and $\underline{W} = \{w_q; \, q = 1, \ldots, Q\}$ are the membership integer vector describing the sub-array configuration of the array elements and the set of sub-array weights, respectively. Moreover, $\beta_m = -\beta_{-m}$, $m = 1, ..., M$, are the excitations affording the optimal/reference difference pattern, and $\delta_{c_m q}$ is the Kronecker delta function (i.e., $\delta_{c_m q} = 1$ if $c_m = q$ and $\delta_{c_m q} = 0$ otherwise).

As regards to the sub-optimal coefficients b_m, $m = 1, ..., M$, affording the compromise difference pattern in (2), they are defined as $b_m = \alpha_m \delta_{c_m q} w_q$, $m = 1, ..., M$. It is simple noticing that (3) defines a least square problem where, for a given configuration $\underline{C}$, the set of sub-array weights minimizing (3) is unequivocally determined by imposing

$$\left. \frac{\partial \Psi \left(\underline{C}, \, \underline{W} \right)}{\partial w_q} \right|_{\underline{C}} = 0, \quad q = 1, ..., Q. \tag{4}$$

Accordingly, it turns out that the sub-array weights can be computed as the weighted arithmetic mean (with weights α_m^2) of the optimal parameters $\frac{\beta_m}{\alpha_m}$ [11]:

$$w_q \left(\underline{C} \right) = \frac{\sum_{m=1}^{M} \alpha_m^2 \left(\frac{\beta_m}{\alpha_m} \delta_{c_m q} \right)}{\sum_{m=1}^{M} \alpha_m^2 \left(\delta_{c_m q} \right)} , \quad q = 1, ..., Q. \tag{5}$$

In order to determine the "best" aggregation [i.e., $\underline{C}^{opt} = Min_{\underline{C}} \{\underline{C}, \, \underline{W} \left(\underline{C} \right)\}$, where the array vector $\underline{W} \left(\underline{C} \right)$ is computed as in (5)], which minimizes the functional in (3), the ACO-based procedure described in [13] is used to sample the solution space of admissible element clustering mapped into a suitable graph representation [11].

Although computationally efficient and reliable, the main drawback of such a method lies in the impossibility of having "*an individual control of the sidelobe levels of final patterns*" [10] because of the excitation matching nature of the underlying mathematical formulation. In order to overcome this drawback and instead of using Eq. (5), the sub-array weights are determined minimizing the following convex function [10]

$$\Psi^{CP} \left(\underline{W} \right) = \left. \frac{d Re \left\{ AF^d \left(\theta \right) \right\}}{d\theta} \right|_{\theta = 0^o}, \tag{6}$$

subject to a set of non-negative constraints on the power pattern $\left| AF^d \left(\theta \right) \right|^2 \leq UB \left(\theta \right)$, still keeping the ACO-defined aggregation, $\underline{C}^{opt} = \underline{C}^{ACO}$. More specifically, the function $UB \left(\theta \right)$ defines an upper bound mask and Re denotes the real part.

As regards to the convex programming minimization, the iterative process starts at $k = 0$ from the guess solution defined by $\underline{C} = \underline{C}^{ACO}$ and $\underline{W}^{(0)} = \underline{W}^{ACO} = \left\{ w_q^{ACO} \left(\underline{C}^{ACO} \right), \, q = 1, ..., Q \right\}$

and standard $MATLAB$ subroutines are used [19].

3   Numerical Results

In order to show the potentialities of the proposed hybrid approach, some preliminary results are discussed and compared with the solutions obtained by means of another hybrid approach presented in [16] as well as from single-step (unconstrained) excitation matching approaches based on the minimization of (3).

The first example (*Experiment 1*) is concerned with a linear antenna of $N = 20$ elements spaced by $d = \frac{\lambda}{2}$. The sum channel provides a Dolph-Chebyshev pattern with $SLL = -25\,dB$ [17], while $Q = 8$ sub-arrays are considered to generate the compromise pattern whose reference difference excitations afford a Zolotarev difference pattern with $SLL = -40\,dB$ [18]. Starting from a uniform distribution of the array elements in each sub-array, the Border Element Method (BEM) (i.e., a local search technique proposed in [11]) gets stuck into a local minimum of the solution space of $U = 36$ admissible aggregations after $k = K_{end}^{BEM} = 3$ iterations (k being the iteration index). The corresponding solution presents a cost function value equal to $\Psi\left(\underline{C}^{BEM}\right) = 2.49 \times 10^{-4}$. On the other hand, the ACO-based procedure has succeeded in finding the global minimum of the matching problem in $K_{end}^{ACO} = 2$ iterations [$\Psi\left(\underline{C}^{ACO}\right) = 1.13 \times 10^{-5}$] with a colony of only $I = 5$ ants, $H = 0.1$ and $\rho = 0.05$ [13] being the ACO parameters setting. The synthesized sub-array configurations are $\underline{C}^{BEM} = \{1\,2\,3\,5\,7\,8\,6\,4\,2\,1\}$ and $\underline{C}^{ACO} = \{1\,3\,5\,7\,8\,8\,7\,6\,4\,2\}$ and the corresponding sub-array weights obtained through Eq. (5) are $\underline{W}^{BEM} = \{0.21,\ 0.61,\ 0.92,\ 0.98,\ 1.16,\ 1.18,\ 1.28,\ 1.29\}$ and $\underline{W}^{ACO} = \{0.20,\ 0.24,\ 0.59,\ 0.73,\ 0.92,\ 0.98,\ 1.17,\ 1.28\}$.

In order to complete the two-step hybrid strategy, the convex programming procedure has been used to minimize the peak sidelobe starting from the sub-array configurations found at the end of the first step. The values of the sub-array weights synthesized by means of the hybrid approaches are $\underline{W}^{Hyb-BEM} = \{2.11,\ 5.85,\ 8.51,\ 8.99,\ 10.78,\ 11.05,\ 12.10,\ 12.04\}$ and $\underline{W}^{Hyb-ACO} = \{2.18,\ 2.44,\ 6.26,\ 7.39,\ 9.71,\ 10.06,\ 12.15,\ 13.46\}$. The arising patterns together with those obtained with the one-step excitation matching approaches (i.e., BEM and ACO) are shown in Fig. 1 with a detail of the behavior of the secondary lobes. Both hybrid approaches outperform their corresponding single-step counterpart in terms of SLL minimization

(Tab. I). Such an event further confirms the importance of exploiting the partial convexity of the problem at hand [10][15]. As far as the $Hybrid - ACO$ is concerned, the convergence pattern faithfully matches the reference one and outperforms the solution of the $Hybrid - BEM$ [15] as well as that of the "bare" ACO of more than $3.5\,dB$ and $2.4\,dB$, respectively.

For completeness, the plots of the cost function (6) and of the index C_k^θ during the iterative optimization are shown in Fig. 2, where $C_k^\theta = max_\theta \left\{ AF_k^d (\theta) - UB (\theta) \right\}$, $\theta \in \left[-\frac{\pi}{2}; \frac{\pi}{2} \right]$, $UB(\theta)$ being set to $-40\,dB$ below the peak of the main lobe in the angular range $|\theta| \geq 14.5^o$. As it can be noticed, the $Hybrid - ACO$ turns out to be more effective than the $Hybrid - BEM$ both in minimizing (6) and fitting the pattern constraints $[AF_k^d (\theta) \rightarrow UB (\theta)]$.

As regards to the computational efficiency of the methods at hand, the optimal values of the cost function for each approach, Ψ, the number iterations required until convergence, K_{end}, the mismatch between the synthesized pattern and the user-defined constraint, $C_{K_{end}}^\theta$, and the CPU-time, T, needed to get the final solution on a $3\,GHz$ PC with $2\,GB$ of RAM are reported in Tab. II. As expected, because of the constrained optimization problem at hand, the solution of the quadratic programming problem is much more computationally expensive than that of the corresponding (unconstrained) excitation matching one (Tab. II).

In the second example (*Experiment 2*), a larger array with $N = 40$ elements ($d = \frac{\lambda}{2}$) is considered. Unlike the previous example, the number of sub-arrays is reduced from $Q = 8$ to $Q = 4$. The sum excitations have been set to those of a Dolph-Chebyshev pattern with $SLL = -25\,dB$ [17], while the reference difference excitations have been chosen to afford a Zolotarev difference pattern with $SLL = -30\,dB$ [18]. As regards to the number of possible sub-array configurations, the dimension of the solution space defined through the excitation matching procedure [11] contains $U = 969$ different compromise solutions. As regards to the ACO, a colony of $I = 10$ ants has been used by keeping unaltered the remaining ACO parameters.

As far as the excitation matching (or single-step) approaches are concerned, the BEM and the ACO converge to their final solutions (Tab. III) within $K_{end}^{BEM} = 21$ and $K_{end}^{ACO} = 34$ iterations, respectively. Moreover, the convergence cost function values turns out to be equal to $\Psi \left(\underline{C}^{BEM} \right) = 5.48 \times 10^{-3}$ and $\Psi \left(\underline{C}^{ACO} \right) = 5.00 \times 10^{-3}$. Although the close values of Ψ at the convergence, it is worth noting that the BEM is once again trapped into a local minimum of (3).

As far as the hybrid procedures are concerned, the mask on the secondary lobes has been set to $UB(\theta) = -25\,dB$ below the peak value for $|\theta| \geq 6.3^o$. The synthesized patterns and the equivalent compromise difference excitations are shown in Fig. 3 and Fig. 4, respectively. Analogously to the previous experiment, the importance of avoiding local minima and exploiting the potentialities of an hybrid approach are further underlined by values of the pattern indexes in Tab. IV. More specifically, it is worth pointing out that when the BEM fails in retrieving the optimal sub-array configuration, the successive application of the convex programming procedure cannot be fully/profitably exploited. As a matter of fact, the solution synthesized by means of the "bare" ACO approach outperforms the one with the $Hybrid - BEM$ in terms of SLL minimization (i.e., $SLL^{Hyb-BEM} = -22.60\,dB$ vs. $SLL^{ACO} = -22.93\,dB$), although no-constraints on the SLL have been imposed. Furthermore, the ACO allows a non-negligible computational saving, as pointed out by the values in Tab. V, since it is able to find the final solution almost in real time. On the other hand, the advantages of using an hybrid method clearly appear from the pattern indexes in Tab. IV.

4   Conclusions

In this work, the effectiveness of using hybrid approaches when dealing with the optimal compromise sum-difference problem has been pointed out. It has also been verified that an ACO-based exploration of the solution space for the definition of the sub-array configuration allows one to obtain more effective compromise solutions at the second step of the hybrid procedure where a constrained optimization based on the solution of a convex programming problem is considered.

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FIGURE CAPTIONS

- **Figure 1.** *Experiment 1* ($N = 20$, $Q = 8$) - Compromise difference pattern obtained with the BEM, the ACO and their corresponding hybrid implementations.

- **Figure 2.** *Experiment 1* ($N = 20$, $Q = 8$) - Behavior of the cost function and maximum mismatch value of the power pattern constraints for the hybrid implementation of the BEM and of the ACO.

- **Figure 3.** *Experiment 2* ($N = 40$, $Q = 4$) - Compromise difference pattern obtained with the BEM, the ACO and their corresponding hybrid implementations.

- **Figure 4.** *Experiment 2* ($N = 40$, $Q = 4$) - Optimal (Zolotarev, $SLL = -30\,dB$ [18]) and compromise difference excitations obtained with the BEM, the ACO and their corresponding hybrid implementations together with the reference .

TABLE CAPTIONS

- **Table I.** *Experiment 1* ($N = 20$, $Q = 8$) - Pattern indexes.

- **Table II.** *Experiment 1* ($N = 20$, $Q = 8$) - Computational indexes.

- **Table III.** *Experiment 2* ($N = 40$, $Q = 4$) - Sub-array configurations and weights.

- **Table IV.** *Experiment 2* ($N = 40$, $Q = 4$) - Pattern indexes.

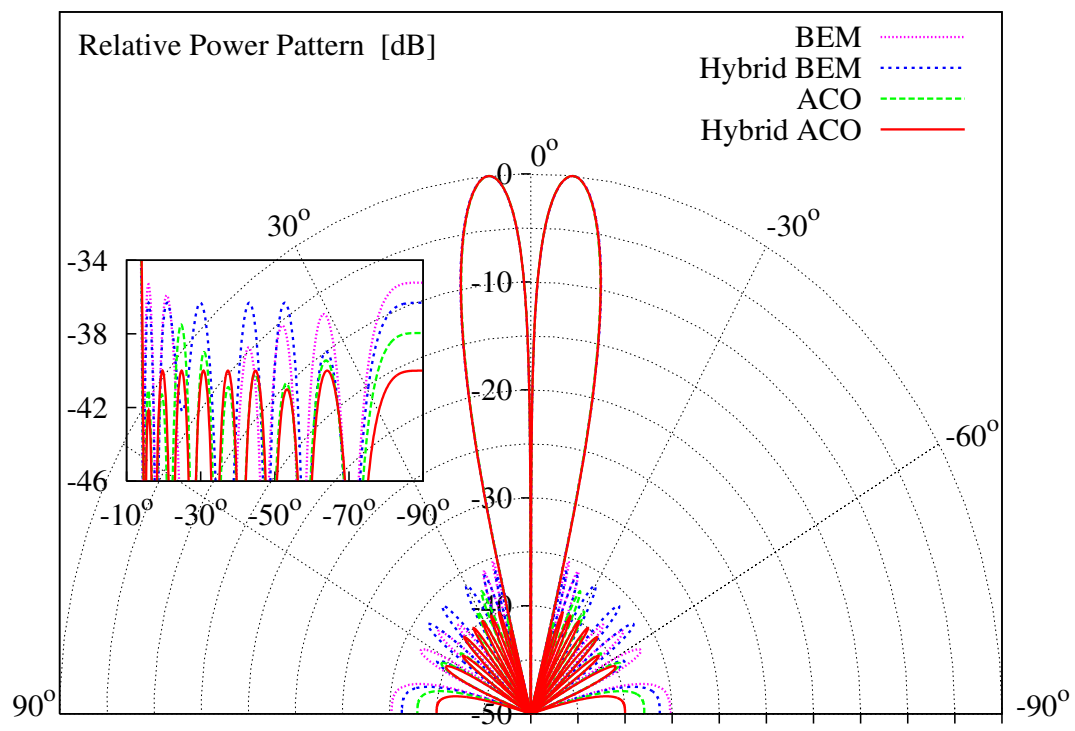- **Table V.** *Experiment 2* ($N = 40$, $Q = 4$) - Computational indexes.

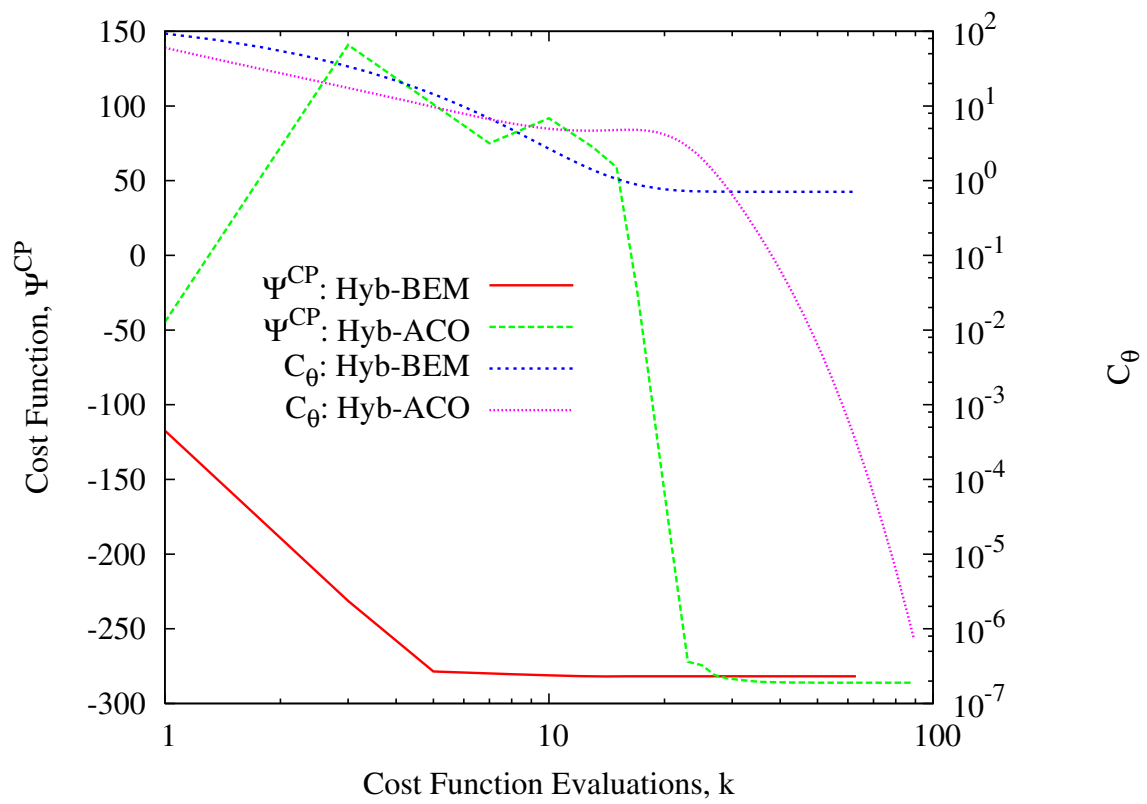**Fig. 1 - P. Rocca *et al.,*** "Ant Colony Based Hybrid Approach ..."

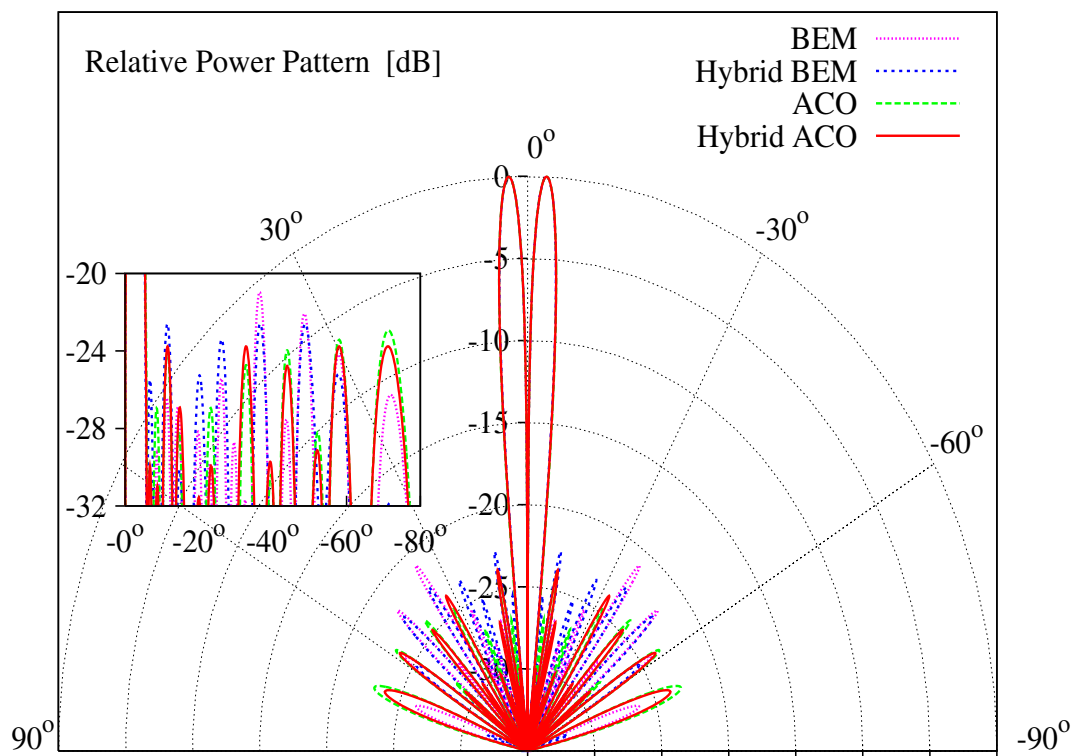**Fig. 2 - P. Rocca *et al.*,** "Ant Colony Based Hybrid Approach ..."

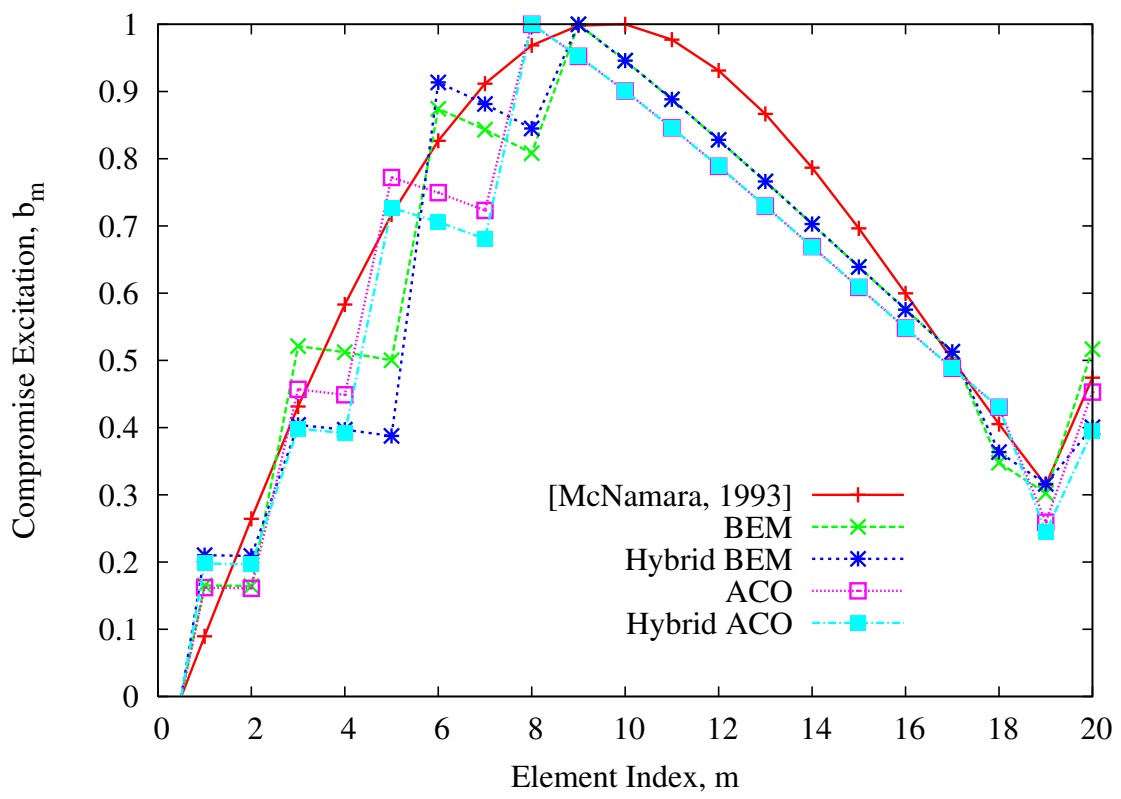**Fig. 3 - P. Rocca** *et al.*, "Ant Colony Based Hybrid Approach ..."

Fig. 4 - P. Rocca *et al.*, "Ant Colony Based Hybrid Approach ..."

$Approach$	$BW\ [deg]$	$SLL\ [dB]$	$D_{max}\ [dB]$
BEM	5.73	-35.22	9.89
$Hybrid\ BEM$	5.73	-36.31	9.89
ACO	5.67	-37.50	9.94
$Hybrid\ ACO$	5.70	-39.92	9.92
$Reference$ [18]	5.67	-40.00	9.94

Tab. I - P. Rocca *et al.,* "Ant Colony Based Hybrid Approach ..."

$Approach$	Ψ	$C^{\theta}_{K_{end}}$	K_{end}	$T\ [sec]$
BEM	2.49×10^{-4}	$-$	3	$< 10^{-8}$
$Hybrid - BEM$	-281.81	7.1×10^{-1}	$3 + 63$	6.305
ACO	1.13×10^{-5}	$-$	2	$< 10^{-8}$
$Hybrid - ACO$	-286.04	7.5×10^{-7}	$2 + 89$	6.775

Tab. II - P. Rocca *et al.,* "Ant Colony Based Hybrid Approach ..."

Approach	$\underline{C}$			
BEM	$\{1\,1\,2\,2\,2\,3\,3\,3\,4\,4\,4\,4\,4\,4\,4\,3\,3\,2\}$			
ACO	$\{1\,1\,2\,2\,3\,3\,3\,4\,4\,4\,4\,4\,4\,4\,4\,4\,3\,2\}$			
	w_1	w_2	w_3	w_4
BEM	0.18	0.57	1.02	1.33
Hybrid BEM	0.15	0.28	0.69	0.86
ACO	0.18	0.51	0.90	1.29
Hybrid ACO	0.16	0.33	0.63	0.96

Tab. III - P. Rocca *et al.,* "Ant Colony Based Hybrid Approach ..."

$Approach$	$BW\ [deg]$	$SLL\ [dB]$	$D_{max}\ [dB]$
BEM	2.50	-20.96	13.39
$Hybrid\ BEM$	2.50	-22.60	13.38
ACO	2.52	-22.93	13.36
$Hybrid\ ACO$	2.52	-23.76	13.37
$Reference$ [18]	2.52	-30.00	13.41

Tab. IV - P. Rocca *et al.,* "Ant Colony Based Hybrid Approach ..."

Approach	Ψ	$C_{K_{end}}^{\theta}$	K_{end}	$T\ [sec]$
BEM	5.48×10^{-3}	$-$	21	$\simeq 10^{-7}$
Hybrid BEM	-85.44	5.80×10^{-7}	$21 + 146$	27.273
ACO	5.00×10^{-3}	$-$	34	
Hybrid ACO	-97.60	2.02×10^{-7}	$34 + 78$	

Tab. V - P. Rocca *et al.,* "Ant Colony Based Hybrid Approach ..."