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ON THE SYNTHESIS OF SUB-ARRAYED PLANAR ARRAY ANTENNAS FOR TRACKING RADAR APPLICATIONS

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January 2011

Technical Report # DISI-11-045

On the Synthesis of Sub-arrayed Planar Array Antennas for Tracking Radar Applications

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Abstract

The synthesis of compromise sum and difference patterns of large planar arrays is addressed in this letter by means of a suitable implementation of the *Contiguous Partition Method* (CPM). By exploiting some properties of the solution space, the generation of compromise sum-difference patterns is recast as the searching of the optimal path in a graph that codes the admissible solution space. Some numerical experiments are provided in order to assess the effectiveness of the proposed method.

Index Terms

Monopulse Antennas, Large Planar Arrays, Compromise Pattern Synthesis, Sum and Difference Beams.

I. INTRODUCTION

Search-and-track systems based on monopulse principles require antennas able to simultaneously provide (on receive) sum and difference patterns. In real world applications, such antennas are usually highly directive with narrow beams (beamwidth B_w typically of the order of 1^o in each angular direction) and low sidelobe levels (SLLs). Moreover, the difference pattern is required to have the slope at boresight as deep as possible to improve the radar sensitivity. In order to fit these requirements, solutions based on planar arrays of wide dimensions with large numbers of elements are usually adopted [1][2]. In this case, complex circuitry is needed to generate three independent beams (i.e., a sum pattern and two orthogonal difference patterns) with greater costs and an enhancement of the mutual electromagnetic interferences. In order to avoid such drawbacks, the sub-arraying strategy has been proposed [3]-[9]. Although ill-conditioning does not affect global optimization-based method, the computational burden raises exponentially with the number of elements and it turns out to be a cumbersome penalty in the synthesis of large two-dimensional ($2D$) arrays. As a consequence, the synthesis of planar arrays has been previously addressed in a few works. More in detail, the synthesis of the three monopulse modes of stripline-fed slot arrays and the problem of mutual coupling effects have been considered in [10] and [11], respectively. A method to improve in a particular azimuthal sector the difference radiation pattern sidelobe level of a monopulse antenna of a corporate-fed array type is presented in [12]. Successively, an improved sub-arraying method has been investigated in [4]. The synthesis of planar arrays has been also addressed by means of a Simulated Annealing (SA) algorithm even though for assigned (i.e., not involved in the optimization) sub-array configurations. Unfortunately, only small structures often not adequate for practical applications have been considered.

Recently, a computationally effective strategy has been presented in [9], namely the contiguous partition method (CPM), which takes definite advantage from the knowledge of the reference or optimal difference excitations. As a positive consequence, the CPM guarantees fast convergence to the solution also in facing high-dimensional problems (i.e., with a large number of unknowns) as shown in [13] dealing with linear arrays. Moreover, such a method demonstrated its robustness as well as an easy implementation. In order to evaluate the validity of the underlying idea and to further assess the flexibility of the CPM, the approach is applied here to the synthesis of large $2D$ planar arrays with a large number ($N > 1000$) of radiating elements. On the other hand, it should be pointed out that this work deals with an excitation matching problem (i.e., the definition of a "best compromise" difference pattern close as much as possible to the reference one) and not the SLL control of the achieved solution. As a matter of fact, the CPM, in its bare version, does not allow a direct control of such a parameter. The potentiality of a modified version of the CPM in effectively dealing with the SLL control has been discussed in [14], where the reference difference pattern is updated until the constraints on the compromise solution were satisfied.

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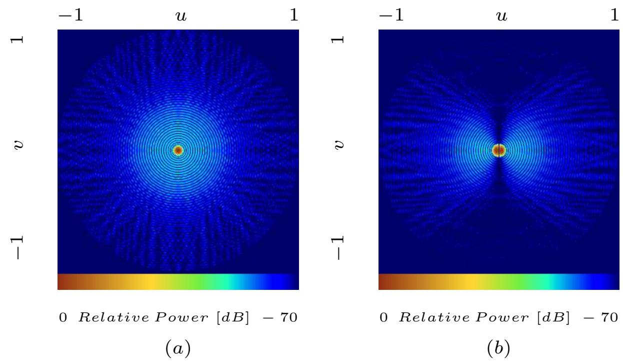


Fig. 1. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - Relative power distribution of the reference (*a*) Taylor sum pattern ($SLL = -50\,dB$, $\overline{n} = 20$) and of the (*b*) $H-mode$ Bayliss difference pattern ($SLL = -50\,dB$, $\overline{n} = 18$) .

II. Mathematical Formulation

Let us consider a planar array with N elements uniformly-spaced on an aperture (d being the inter-element distance along the x and y axes) that generates the following array pattern:

$$AF(\theta,\phi) = \sum_{m=-M}^{M}\sum_{p=-P_m}^{P_m} I_{mp} e^{j\frac{2\pi\sin\theta}{\lambda}(\cos\phi x_m + \sin\phi y_p)} \qquad (1)$$

I_{mp} ($m, p \neq 0$) being an excitation coefficient and $N = \sum_{m=-M}^{M} P_m$. Moreover, $x_m = \left[m - \frac{sgn(m)}{2}\right] \times d$, $m = \pm 1, ..., \pm M$ and $y_p = \left[p - \frac{sgn(p)}{2}\right] \times d$, $p = \pm 1, ..., \pm P_m$.

The reference sum pattern and the difference ones (i.e., the E-mode and the H-mode) are generated by setting the array excitations $\underline{I} = \{I_{mp};\ m = 1, ..., M;\ p = 1, ..., P_m\}$ to $\underline{S} = \left\{s_{mp} = s_{(-m)p} = s_{m(-n)} = s_{(-m)(-p)};\ m = 1, ..., M;\ p = 1, ..., P_m\right\}$ and to $\underline{D}^{\triangle} = \left\{d_{mp}^{\triangle} = d_{(-m)p}^{\triangle} = -d_{m(-p)}^{\triangle} = -d_{(-m)(-p)}^{\triangle};\ m = 1, ..., M;\ p = 1, ..., P_m\right\}$, $\triangle = E, H$, respectively. The above assumed quadrantal symmetry or anti-symmetry allows one to consider only $N_r = \frac{N}{4}$ excitations during the synthesis process. However, since the implementation of three totally independent signal feeds is generally out of the question, the optimal compromise technique is adopted. Such a method consists in first fixing the element excitations affording the optimal sum pattern (i.e., $\underline{I} = \underline{S}$) and then determining the best partition of the N_r array elements in Q sub-arrays (i.e., the aggregation vector $\underline{A}^{\triangle} = \left\{a_{mp}^{\triangle};\ m = 1, ..., M;\ p = 1, ..., P_m\right\}$, where $a_{mp}^{\triangle} \in [1, Q]$) and the sub-array weights $\underline{W}^{\triangle} = \left\{w_q^{\triangle};\ q = 1, ..., Q\right\}$ such that the difference patterns $AF = AF\left\{\underline{C}^{\triangle}\right\}$, $\triangle = E, H$, generated by the compromise excitations $\underline{C}^{\triangle} = \left\{c_{mp}^{\triangle} = s_{mp}\delta\left(a_{mp}^{\triangle}, q\right) w_q^{\triangle};\ m = 1, ..., M;\ p = 1, ..., P_m\right\}$[1] approximate as closely as possible the reference ones, $AF = AF\left\{\underline{D}^{\triangle}\right\}$.

Towards this end and likewise the linear case [9], a suitable customization of the CPM technique is adopted for the two-dimensional architecture, as well. In the following, the key-points of such an implementation will be detailed also pointing out the main differences with respect to the case of linear arrays.

Starting from the observation [9] that the compromise solution is a *contiguous partition* (CP) of the ordered list $\underline{L} = \{l_n;\ n = 1, ..., N_r\}$, $l_n \leq l_{n+1}$ ($n = 1, ..., N_r - 1$), $l_1 = min_{mp}\left\{\gamma_{mp}^{\triangle}\right\}$, $l_{N_r} = max_{mp}\left\{\gamma_{mp}^{\triangle}\right\}$, $\gamma_{mp}^{\triangle}$ being the *reference gain* defined as $\gamma_{mp}^{\triangle} = \frac{d_{mp}^{\triangle}}{s_{mp}}$, the solution space (i.e., the whole set of CPs) is coded into a suitable graph to minimize the storage costs as well as to facilitate the sampling of the space of admissible solutions. As a matter of fact, the use of the tree-based representation of the linear case would have required a non-negligible amount of computer memory and a redundant description with some portions of the tree recursively-shared. The graph is composed by Q rows and N_r columns. The q-th row is related to the q-th sub-array ($q = 1, ..., Q$), whereas the n-th column ($n = 1, ..., N_r$) maps the l_n-th element of $\underline{L}$. A path ψ of the graph codes a compromise solution and it is constituted by a set of N_r vertexes, $\{t_n;\ n = 1, ..., N_r\}$, connected by $N_r - 1$ links, $\{e_n;\ n = 1, ..., N_r - 1\}$.

[1] $\delta\left(a_{mp}^{\triangle}, q\right) = 1$ if $a_{mp}^{\triangle} = q$ and $\delta\left(a_{mp}^{\triangle}, q\right) = 0$, otherwise.

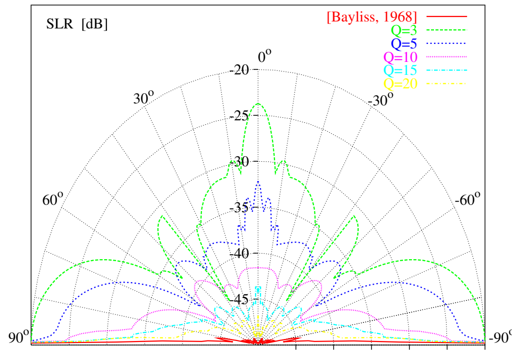


Fig. 2. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - Polar plots of the synthesized SLR values in the range $\phi \in [0^o, 89^o]$ when $Q = 3, 5, 10, 15, 20$ (Reference Bayliss pattern: $SLL = -50\,dB$, $\overline{n} = 18$).

The optimal compromise corresponds to the aggregation $\underline{A}^{\triangle}_{opt}$ that minimizes the cost function

$$\Psi\left(\underline{A}^{\triangle}\right) = \frac{1}{N_r}\sum_{q=1}^{Q}\sum_{m=1}^{M}\sum_{p=1}^{P_m} s^2_{mp}\left|\left[\gamma^{\triangle}_{mp} - g_{mpq}\left(\underline{A}^{\triangle}\right)\right]\right|^2, \tag{2}$$

which quantifies the distance between the reference excitations and the compromise ones, $g^{\triangle}_{mnq} = g_{mnq}\left(\underline{A}^{\triangle}\right)$ being the *estimated gains* given by

$$g^{\triangle}_{mpq} = \frac{\sum_{m=1}^{M}\sum_{p=1}^{P_m} s^2_{mp}\delta\left(a^{\triangle}_{mp}, q\right)\gamma^{\triangle}_{mp}}{\sum_{m=1}^{M}\sum_{p=1}^{P_m} s^2_{mp}\delta\left(a^{\triangle}_{mp}, q\right)}, \quad m = 1, ..., M;\ \ p = 1, ..., P_m;\ \ q = 1, ..., Q. \tag{3}$$

In order to determine $\underline{A}^{\triangle}_{opt}$, a sequence of trial solutions $\left\{\underline{A}^{\triangle}_{k};\, k = 1, ..., k_{end}\right\}$ or, in an equivalent fashion, paths of the graph $\{\psi_k;\, k = 1, ..., k_{end}\}$ is generated by exploring the graph structure, k being the iteration index. The initial path $\psi_0 = \left\{\left(t^{(k)}_n, e^{(k)}_m\right); n = 1, ..., N_r;\, m = 1, ..., N_r - 1\right\}$ is generated by setting $arg\left(t^{(0)}_1\right) = 1$ and $arg\left(t^{(0)}_N\right) = Q$ and randomly assigning the other vertexes to the sub-arrays such that $arg\left(t^{(0)}_{n-1}\right) \leq arg\left(t^{(0)}_{n}\right) \leq arg\left(t^{(0)}_{n+1}\right)$ and there is an uniform distribution of the array elements among the sub-arrays. Then, the trial path ψ_k is iteratively updated ($\psi_k \leftarrow \psi_{k+1}$, $\underline{A}^{\triangle}_{k} \leftarrow \underline{A}^{\triangle}_{k+1}$) just modifying the memberships of the *border vertexes*[2] of ψ_k and the corresponding links, until a maximum number of iterations K_{max} ($k > K_{max}$) or the following stationary condition holds true. The solution reached at $k = k_{end}$ (i.e., the path ψ_k and the corresponding aggregation $\underline{A}^{\triangle}_{k}$) is assumed as optimal compromise and used to define the sub-array weights as follows

$$w^{\triangle}_{q} = \delta\left(a^{\triangle}_{mp}, q\right) g^{\triangle}_{mpq} \quad m = 1, ..., M;\ \ p = 1, ..., P_m;\ \ q = 1, ..., Q. \tag{4}$$

III. Numerical Assessment

This section is devoted to assess the reliability and efficiency of the CPM in synthesizing wide planar arrays composed by large numbers of radiating elements. As an illustrative test case, let us consider a planar geometry with circular boundary and radius $r = 20\lambda$. The $N = 7860$ radiating elements are displaced on a regular grid $\frac{\lambda}{2}$-spaced along the two Cartesian directions. Concerning the optimal patterns, the sum excitations $\underline{S}$ have been fixed to those of the Taylor pattern [15] with $SLL = -50\,dB$ and $\overline{n} = 20$ [Fig. 1(a)][3], whereas the reference $H-mode$ $\underline{D}^H$ has been chosen to afford a Bayliss pattern [15] with $SLL = -50\,dB$ and $\overline{n} = 18$ [Fig. 1(b)]. The beamwidths of the sum and difference patterns are equal to $B^S_w = 1.57^o$ and $B^{D^H}_w = 1.26^o$, respectively. Because of the aperture geometry, the optimization has been limited to the difference $H - mode$ since the $E-mode$ excitations satisfy the following relationship $\underline{B}^E = \left\{b^E_{mp} = -b^H_{mp};\, m = 1, ..., M;\, p = 1, ..., P_m\right\}$. Such a

[2]A vertex t_n ($n = 2, ..., N - 1$) is called *border vertex* when it has at most one of its adjacent vertexes, t_{n-1} or t_{n+1}, that belongs to a different row of the graph.

[3]In the figures, $u = \sin\theta\cos\phi$ and $v = \sin\theta\sin\phi$ [15], where $\theta \in [0, \pi/2]$ and $\phi \in [0, 2\pi]$.

TABLE I

VALUES OF THE PATTERN INDEXES.

$[dB]$	$Q=3$	$Q=5$	$Q=10$	$Q=15$	$Q=20$	$Ref.$ [15]
$SLL\,[dB]$	-23.72	-32.19	-41.62	-43.79	-46.81	-50.00
$B_w\,[deg]$	1.251	1.233	1.229	1.228	1.224	1.224

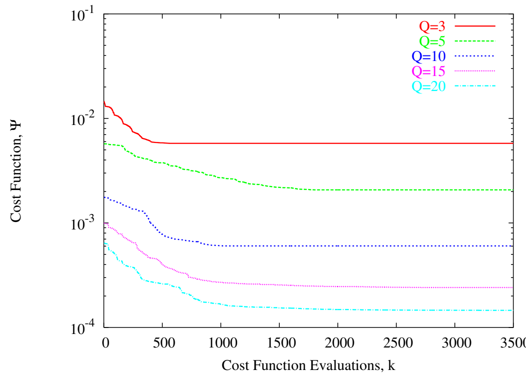
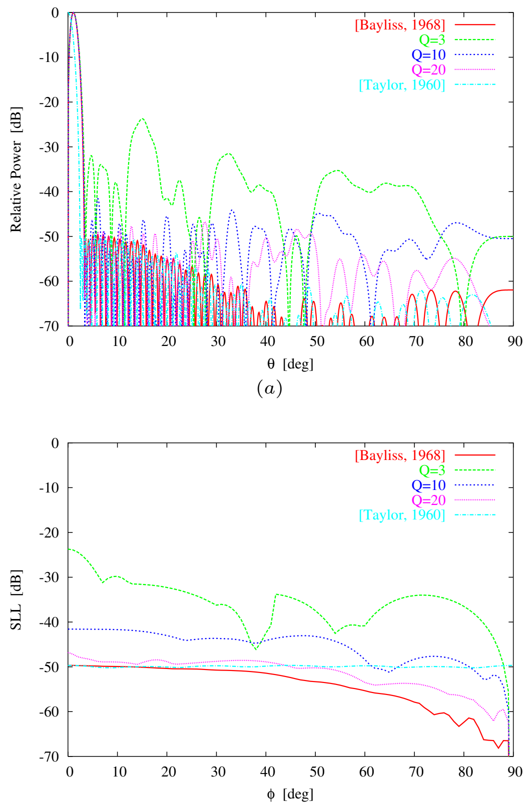


Fig. 3. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - Behavior of the cost function Ψ versus the iteration index k.

condition allows one to synthesize a radar antenna with the same angular resolution in both the azimuthal (H) and elevation (E) directions. On the other hand, it should be noticed that the CPM might be applied twice and independently for the two difference modes to obtain different performances along each angular coordinate without a significant increasing of the computational costs. As far as the sub-arraying strategy is concerned, the number of sub-arrays of the compromise feed network has been varied in the range $Q = [3, 20]$. Moreover, besides the $-3\,dB$ beamwidth B_w, let us consider the sidelobe ratio (SLR) as a quantitative index to evaluate the sidelobe features of the synthesized pattern in the whole aperture. It is defined as follows

$$SLR\,(\phi) = \frac{SLL\,(\phi)}{max_\theta\,[AF\,(\theta,\phi)]},\ \ 0 \leq \theta < \frac{\pi}{2} \tag{5}$$

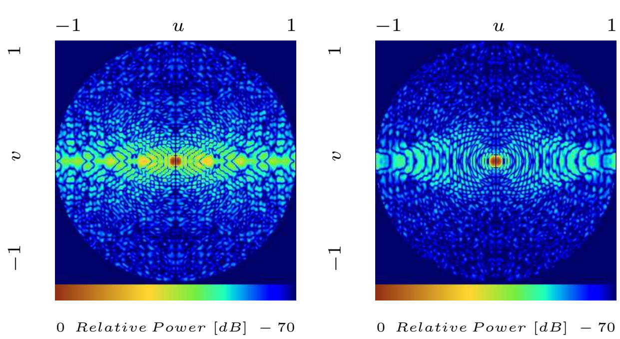
$AF\,(\theta,\phi)$ being the array factor. Since the difference $H - mode$ vanishes at $\phi = 90^o$, the values of the SLR of the synthesized patterns have been controlled in the range $\phi \in [0^o, 89^o]$. Fig. 2 shows the plots of the SLRs to fully evaluate the CPM behavior when $Q = 3,\ 5,\ 10,\ 15,\ 20$. For completeness, the values of the maximum level of the secondary lobes on the whole aperture and the $-3\,dB$ B_w are reported in Tab. I. As expected, the CPM guarantees to asymptotically approximate the reference pattern when the number of sub-arrays gets closer and closer to N_r. Such a property is further confirmed by the behavior of the cost function Ψ (Fig. 3), which quantifies the fitting of the compromise excitations with the reference ones. These plots point out the robustness and effectiveness of the proposed method in matching the reference pattern. As a matter of fact, ever since the initial iteration ($k = 0$) when an uniform partitioning of the ordered list $\underline{L}$ is chosen, the solution appears to be closer and closer to the reference one just increasing the number of sub-arrays (Fig. 3, $k = 0$). Moreover, for a given value of Q, the CPM better approximates the Bayliss pattern iteratively ($k \geq 1$) changing the sub-array memberships of the border elements. Fig. 4(a) and Fig. 4(b) give the plots of the u-cuts at $\phi = 0^o$ and the pictorial representations of the SLL behavior, respectively, of the compromise solutions synthesized by the CPM as well as those of the optimal patterns. Moreover, the relative power distributions obtained at the convergence iteration ($k = k_{end}$) when $Q = 3$ and $Q = 10$ are shown in Fig. 5 . In order to allow the reproduction of those patterns, to be also used as benchmarks in future comparisons, Fig. 6 and Tab. II give a pictorial representation of the sub-array configurations and the values of the sub-array gains, respectively. Finally, since a key feature of the proposed technique is the faster convergence, let us focus on the CPM computational efficiency by analyzing the values of the indexes reported in Tab. III . More in detail, k_{end} is the number of cost function evaluations to reach the final solution, T is the corresponding CPU-time. Moreover, U and $U^{(ess)}$ indicate the dimension of the solution space of the stochastic optimization-based approaches and of the CPM, respectively. Due to the non-negligible reduction of



(a)

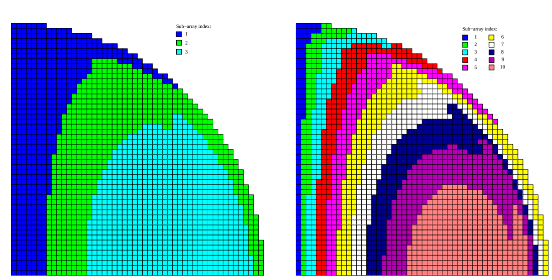
(b)

Fig. 4. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - (a) Azimuthal ($\phi = 0^o$) plot of the relative power and (b) behavior of the SLL versus the azimuth angle for the Bayliss pattern ($SLL_{ref} = -50\,dB$, $\overline{n} = 18$), the synthesized ones with $Q = 3$, 10, 20 sub-arrays, and the Taylor pattern ($SLL = -50\,dB$, $\overline{n} = 20$).



(a) (b)

Fig. 5. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - Relative power distribution of the difference $H - mode$ pattern when (a) $Q = 3$ and (b) $Q = 5$.



(a) (b)

Fig. 6. *Sub-Arrayed Planar Array Synthesis* ($N = 7860$, $d = \frac{\lambda}{2}$, $r = 20\lambda$) - Sub-array configuration of the difference $H - mode$ pattern when (a) $Q = 3$ and (b) $Q = 5$.

TABLE II

SUB-ARRAY GAINS FOR THE SOLUTION WITH $Q = 3$ AND $Q = 10$.

Q	$w_1, \ldots, w_q$; $q = 1, \ldots, Q$
3	0.288, 0.870, 1.484
10	0.041, 0.135, 0.258, 0.421, 0.528, 0.795, 1.009, 1.230, 1.462, 1.711

TABLE III

VALUES OF THE COMPUTATIONAL INDEXES.

	k_{end}	T $[sec]$	U^{ess}	U
$Q = 3$	579	11.56	1.92×10^{6}	$\mathcal{O}\left(10^{937}\right)$
$Q = 5$	1804	33.54	6.18×10^{11}	$\mathcal{O}\left(10^{1373}\right)$
$Q = 10$	1084	20.96	1.17×10^{24}	$\mathcal{O}\left(10^{1965}\right)$
$Q = 15$	2795	24.19	1.35×10^{35}	$\mathcal{O}\left(10^{2311}\right)$
$Q = 20$	3207	48.57	2.79×10^{45}	$\mathcal{O}\left(10^{2556}\right)$

the dimension of the solution space as well as the efficiency of the graph-based searching procedure, the CPU-time to obtain the final solution is less than one minute on a $3.4\,GHz$ PC with $2\,GB$ of RAM , whatever the experiment (Tab. III).

IV. CONCLUSIONS

In this letter, the design of large planar arrays generating compromise sum-difference patterns has been carried by means of the CPM, which exploits the knowledge of the independently optimum sum and difference excitations. Starting from a graph-based representation of the space of admissible solutions, the synthesis of compromise difference modes has been obtained through a path searching procedure that allows a considerable reduction of the problem complexity as well as a significant saving in terms of storage resources and CPU-time.

REFERENCES

[1] S. M. Sherman. *Monopulse principles and techniques*, Artech House, 1984.

[2] M. I. Skolnik. *Radar handbook*, 2nd edn. McGraw-Hill, 1990.

[3] D. A. McNamara, "Synthesis of sub-arrayed monopulse linear arrays through matching of independently optimum sum and difference excitations," *IEE Proc. H*, vol. 135, pp. 293-296, Oct. 1988.

[4] F. Ares, *et al.*, "Optimal compromise among sum and difference patterns," *J. Electromag. Waves and Appl.*, vol. 10, pp. 1543-1555, 1996.

[5] P. Lopez, *et al.*, "Subarray weighting for difference patterns of monopulse antennas: joint optimization of subarray configurations and weights," *IEEE Trans. Antennas Propagat.*, vol. 49, pp. 1606-1608, Nov. 2001.

[6] S. Caorsi, *et al.*, "Optimization of the difference patterns for monopulse antennas by a hybrid real/integer-coded differential evolution method," *IEEE Trans. Antennas Propagat.*, vol. 53, pp. 372-376, Jan. 2005.

[7] A. Massa, *et al.*, "Optimization of the directivity of a monopulse antenna with a subarray weighting by a hybrid differential evolution method," *IEEE Antennas Wireless Propagat. Lett.* , vol. 5, pp. 155-158, 2006.

[8] M. D'Urso, *et al.*, "An effective hybrid approach for the optimal synthesis of monopulse antennas," *IEEE Trans. Antennas Propagat.*, vol. 55, pp. 1059-1066, Apr. 2007.

[9] L. Manica, *et al.*, "An innovative approach based on a tree-searching algorithm for the optimal matching of independently optimum sum and difference excitations," *IEEE Trans. Antennas Propagat.*, vol. 56, no. 1, pp. 58-66, Jan. 2008.

[10] L. Josefsson, *et al.*, "A monopulse flat plate antenna for missile seeker," *Proc. Military Electronics Defence Expo.*, Weisbaden, pp. 334-346, Sep. 1977.

[11] J. H. Schaffner, *et al.*, "Compromises among optimum sum and difference patterns for planar waveguide-fed slot array," *Alta Frequenza*, vol. 50, pp. 312-319, Nov.-Dec. 1981.

[12] A. G. Derneryd, "Optimization of difference pattern sidelobes in the presence of fictitious blockage," *J. Electromag. Waves and Appl.*, vol. 2, no. 5/6, pp. 535-543, 1988.

[13] P. Rocca, *et al.*, "Synthesis of large monopulse linear arrays through a tree-based optimal excitations matching," *IEEE Antennas Wireless Propagat. Lett.* , vol. 6, pp. 436-439, 2007.

[14] P. Rocca, *et al.*, "An effective excitation matching method for the synthesis of optimal compromises between sum and difference patterns in planar arrays," *Progress in Electromagnetic Research B*, vol. 3, page 115-130, 2008.

[15] R. S. Elliott. *Antenna Theory and Design*, Wiley-Interscience IEEE Press, 2003.