UNIVERSITY
OF TRENTO

DIPARTIMENTO DI INGEGNERIA E SCIENZA DELL'INFORMAZIONE

38123 Povo – Trento (Italy), Via Sommarive 14
http://www.disi.unitn.it

ADS-BASED HYBRID METHODS FOR ARRAY THINNING

F. Caramanica, M. Seghi, G. Oliveri, and A. Massa

January 2011

Technical Report # DISI-11-171

ADS-Based Hybrid Methods for Array Thinning

Federico Caramanica, Massimo Seghi, Giacomo Oliveri, and Andrea Massa

ELEDIA Research Group, Department of Information Engineering and Computer Science
University of Trento, via Sommarive 14, I-38123, Trento, Italy
federico.caramanica@disi.unitn.it, massimo.seghi@gmail.com, giacomo.oliveri@disi.unitn.it,
andrea.massa@ing.unitn.it

Abstract: This paper presents different Almost Difference Set (ADS)-based hybrid methodologies to design linear thinned arrays. The proposed methods, which are based on genetic algorithms, allow one to overcome the limitations of ADS-based thinned arrays in terms of design flexibility and/or performances. The numerical validation points out the efficiency of the proposed methodologies with respect to standard ADS designs as well as standard stochastic techniques.

Keywords: Thinned Arrays, Almost Difference Sets, Sidelobe Control, Genetic Algorithm.

1. **Introduction**

Large thinned arrays are usually used in modern radars for remote sensing and traffic control, devices for satellite and ground communications, and biomedical imaging systems, since they often require radiation patterns with a very high directivity and a low weight, consumption, hardware complexity, and costs [1]. Unfortunately, thinning large arrays reduces the control of the peak sidelobe level (PSL) [1]. Several techniques have been proposed to overcome this drawback [1-6] and efficient methods for designing thinned arrays with low PSLs are still of great interest [7]. More specifically, the exploitation of Difference Sets (DSs) to analytically determine thinned arrangements with well controlled sidelobes [7] has been recently extended to a wider class of geometries by exploiting the mathematical properties of Almost Difference Sets (ADSs) [8-11], and a-priori bounds for the performances of the synthesized arrays have been provided [8]. However, the use of ADS sequences for array thinning has some limitations [8]. More specifically: (*a*) ADS-based arrays usually provide sub-optimal PSL performances; (*b*) although large repositories of ADSs are available [9], ADS arrays with arbitrary aperture sizes and thinning factors cannot be designed [8]; (*c*) general purpose ADS construction techniques do not exist at present (even for admissible ADS parameters) and ADSs have to determined on a case by case basis using suitable construction theorems [8-11].

In this paper a technique able to enhance the ADS-based design methodology and to overcome the above limitations is proposed. More specifically, a GA-based procedure exploiting and enhancing the approach in [8] is chosen since:

(a) GAs are intrinsically able to deal with binary optimization problems [3][4];
(b) GAs have been successfully applied to the design of thinned arrays [5];
(c) ADS-derived a-priori information can be easily integrated into GAs [6].

Accordingly, a GA-enhanced ADS methodology is hereinafter proposed to overcome the limitations of standard ADS thinning. It should be pointed out that the main objective of the paper is not only to propose a hybrid technique to design thinned arrays, but rather to present a methodological approach to address the limitations of ADS-based arrays when/where either their performances do not comply the radiation requirements of the application at hand or no ADS is available for the geometry (aperture size or thinning factor) under study. In order to focus on that, the proposed method is applied to three different classes of problems related to the main limitations of ADS-based arrays. The outline of the paper is as follows. After a short review on ADS thinning, the GA-enhanced methodology is proposed to address three different problems concerned with ADS-based arrays (Sect. 2). The approach is then validated by means of several numerical simulations. Representative results concerned with both small and large arrays as well as different thinning factors are discussed to point out its reliability (Sect. 3). Finally, some conclusions are drawn (Sect. 4).

2. **Mathematical Formulation and Hybrid Methodology**

Let us consider a linear uniform lattice of N positions spaced by s wavelengths. A thinned array with K active elements defined on such an aperture will exhibit an array factor equal to $S(u)=\sum_{n=0}^{N-1} w(n)\exp[i2\pi(nsu)]$, where $w(n)\in\{0,1\}$ ($n=0,..,N-1$) and $\sum_{n=0}^{N-1} w(n)=K$. According to the technique outlined in [8], the ADS-based thinned array excitations are defined as follows:

$$w(n)=\begin{cases}1 & \text{if } n\in D\\ 0 & \text{otherwise}\end{cases} \tag{1}$$

where D is a (N,K,Λ,t)-ADS, i.e. a set of K unique integers in the range $0,...,N-1$ whose associated sequences exhibit a periodic autocorrelation function, defined as

$$A(\tau)=\sum_{n=0}^{N-1} w(n)w[(n+\tau)\bmod N], \tag{2}$$

with the following three-level behavior [7]

$$A(\tau)=\begin{cases}K & \text{if } \tau=0\\ \Lambda & \text{for } t \text{ values of } \tau\\ \Lambda+1 & \text{otherwise}\end{cases} \tag{3}$$

Thanks (3) and to the relations between the power pattern of the ADS-based linear arrays and the Fourier transform of the associated autocorrelation [i.e. $|S(u_n)|^2=DFT[A(\tau)]$, where $u=\frac{n}{Ns}$ and where *DFT* stands for the discrete Fourier transform operator], it can be shown that the associated PSL complies with the following inequality [8]

$$PSL_{INF}\leq PSL_{MIN}\leq PSL_{OPT}\leq PSL_{MAX}\leq PSL_{SUP} \tag{2}$$

where $PSL_{OPT}\equiv\min_{\sigma}\left[PSL\left(D^{(\sigma)}\right)\right]$ ($\sigma=0,..,N-1$, $PSL(D^{(\sigma)})\equiv\frac{\max_{(u)\notin R}|S(u)|^2}{|S(0)|^2}$, R is the mainlobe region [8], $D^{(\sigma)}$ is the cyclically shifted version of σ positions of the original ADS, and $PSL_{INF}=\frac{K-\Lambda-1-\sqrt{\frac{t(N-t)}{N-1}}}{(N-1)\Lambda+K-1+N-t}$,

$PSL_{SUP}=\left(0.8488+1.128\log_{10}N\right)\frac{K-\Lambda-1+\sqrt{t(N-t)}}{(N-1)\Lambda+K-1+N-t}$, $\Psi=\frac{\min_{k\neq 0}\left\{\left|\sum_{n=0}^{N-1} w(p,q)\exp\left[-2\pi j\left(\frac{nk}{N}\right)\right]\right|^2\right\}}{K^2}$,

$PSL_{MAX} = \Xi(0.8488 + 1.128\log_{10} N)$, $PSL_{MIN} = \Xi$, and $\Xi = \frac{\max_{k\neq 0}\left\{\left|\sum_{n=0}^{N-1} w(n)\exp\left[-2\pi j\left(\frac{nk}{N}\right)\right]\right|^2\right\}}{K^2}$. The ADS-based illustrated methodology does not require any optimization, but only the computation of N array factors (which can be performed by means of FFT procedures) resulting from the cyclic shifts.

Despite its advantages, the outlined ADS-based approach has the above discussed drawbacks. In order to overcome such limitations, a hybrid approach (ADSGA in the following) is proposed. More specifically, the following representative critical situations of the ADS approach are modeled and solved in the following Section by means of suitable optimization problems [ρ being a generic trial solution]:

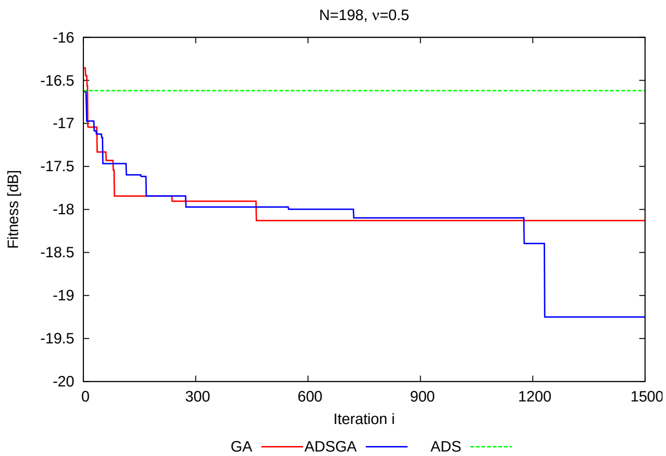
- *Problem A* – minimize $F(\rho) = \frac{\max_{u>U}|S(u)|^2}{|S(0)|^2}$ (U being the location of the first null in the power pattern), subject to $N = N_{ADS}$, $K \leq K_{ADS}$; in this case the objective is to improve the PSL of a reference ADS array without enlarging the number of active elements.
- *Problem B* – minimize $F(\rho) = \frac{\max_{u>U}|S(u)|^2}{|S(0)|^2}$, subject to $N = \hat{N}$, $K \leq \hat{K}$ (with $\hat{N} \neq N_{ADS}$ and/or $\hat{K} \neq K_{ADS}$); in this case the objective is to find a thinned array design with good PSL performances for a set of parameters for which no ADS exists.

As far as the ADSGA methodology is concerned, the standard structure of the GA is modified to exploit the positive key-features of the ADSs. More specifically, the initial population (i = 0, i being the iteration index) is generated by ranking the N shifted versions of a reference ADS according to their PSLs and then setting the chromosomes of half trial solutions to be equal to the first $P/2$ ADSs sequences (P being the dimension of the GA population). Concerning the remaining of the population, the trial solutions are chosen randomly within the range of admissibility of the problem at hand. Such an initialization allows the "transfer" into the GA chromosomes of the good ADS-based schemata also providing a sufficient variability within the population to avoid the stagnation [4]. As regards the GA operators, both crossover and mutation are applied following the standard binary implementations, but also guaranteeing the updated trial solutions be admissible and comply with the problem constraints (e.g., fixed thinning factor).

3. **Numerical Analysis**

In this section the effectiveness of ADSGA in solving Problems I and II is analyzed by means of a set of representative examples. In all cases the crossover probability has been set to 0.9, the mutation probability to 0.01, and the population size is equal to the array length.

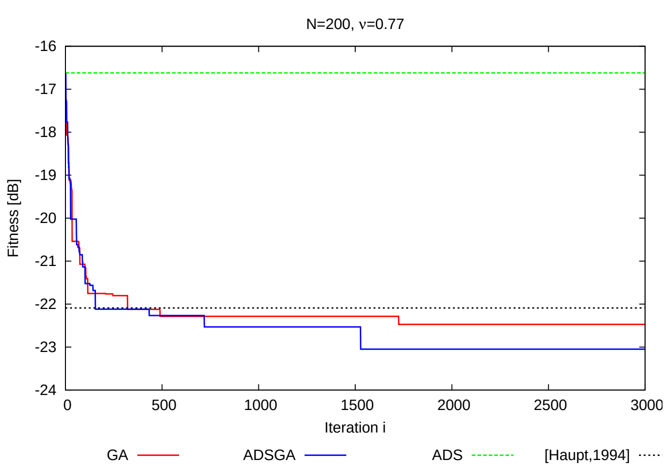
As a first example, Problem A has been solved by considering, as a reference configuration, the $(198,99,49,148)$-ADS [$N_{ADS} = 198$, $K_{ADS} = 99$]. In this case, the PSL performance of the ADS design have been compared with those obtained by a standard GA approach and of the considered ADSGA method. The behavior of the fitness of the optimal trial solution as a function of the iteration number, shown in Fig. 1(*a*), confirms that the proposed ADSGA approach provides improved performances with respect to the ADS methodology in terms of PSL. As a matter of fact, the final ADSGA design exhibit a PSL which is almost 3 dB below that of the starting ADS arrangement [Fig.1(*a*)], without requiring additional elements. Moreover, the ADSGA optimal pattern has a mainlobe size which is close to that of the reference ADS [Fig. 1(*b*)]. Thanks to the *a-priori* information provided by the ADS initialization, moreover, ADSGA outperforms also a standard GA approach in terms of PSL, since the ADSGA obtains a PSL which more than 1 dB below that of GA [Fig. 1(*b*)] and with a comparable speed of convergence [Fig. 1(*a*)].



(*a*) (*b*)

Fig. 1 – *Problem A [* $N=198$, $\nu=0.5$ *].* (*a*) Plots of the optimal fitness versus the iteration number, and (*b*) behaviour of the optimal array pattern of the standard GA, the ADSGA approach, and the ADS method.

As a second example, Problem B has been solved by considering $\hat{N}=200$, $\hat{K}=154$ as reference parameters (the same ADS considered above has been employed for the initialization of the ADSGA). In this case, a reference solution obtained in [3] with the same parameters has been reported in plots as well, for comparison. As it can be noticed from the behavior of the plot as a function of the iteration number [Fig. 2(*a*)], the ADSGA approach outperforms the other considered approaches, improving the PSL of the reference ADS of more than 6 dB. Moreover, it can be pointed out that the ADSGA provides improved performances with respect to the standard GA approach (also in terms of speed of convergence) and to the GA approach discussed in [3], reducing the PSL of the final design without significantly modifying the mainlobe behaviour of the optimal pattern [Fig. 2(*b*)]. Such improved performances are related to the ADS initialization employed in the ADSGA approach, which allows the simple introduction of valuable *a-priori* information in the GA approach to improve its performances.



(*a*) (*b*)

Fig. 2 – *Problem B [* $N=200$, $\nu=0.77$ *].* (*a*) Plots of the optimal fitness versus the iteration number, and (*b*) behaviour of the optimal array pattern of the standard GA, the ADSGA approach, the ADS method, and the solution in [3].

4. **Conclusions**

In this paper, hybrid methodologies have been developed to overcome the limitations (i.e., flexibility and performances) of ADS-based binary sequences for array thinning. The proposed approaches enhance the features of ADS-based designs by exploiting their properties in terms of PSL control and design efficiency. A preliminary numerical analysis has been carried out by addressing different kinds of problems, each one concerned with to a specific ADS limitation. The obtained results have pointed out the performances, reliability and efficiency of the proposed hybrid methodologies.

References

1 C. A. Balanis, *Antenna Theory: Analysis and Design* JohnWiley and Sons, New Jersey, 1997.

2 S. Holm, B. Elgetun, and G. Dahl, "Properties of the beampattern of weight- and layout optimized sparse arrays," *IEEE Trans. Ultrason., Ferroelectr., Freq. Control*, vol. 44, no. 5, pp. 983-991, 1997.

3 R. L. Haupt, "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propag.*, vol. 42, no. 7, pp. 993-999, 1994.

4 R. L. Haupt and D. H. Werner D. H., "*Genetic algorithms in electromagnetics*", JohnWiley and Sons, New Jersey, 2007

5 A. Trucco and V. Murino, "Stochastic optimization of linear sparse arrays," *IEEE J. Ocean Eng.*, vol. 24, no. 3, pp. 291-299, 2004.

6 S. Caorsi, A. Lommi, A. Massa, and M. Pastorino, "Peak sidelobe reduction with a hybrid approach based on GAs and difference sets," *IEEE Trans. Antennas Propag.,* vol. 52, no. 4, pp. 1116-1121, 2004

7 D. G. Leeper, "Isophoric arrays - massively thinned phased arrays with well-controlled sidelobes," *IEEE Trans. Antennas Propag.*, vol. 47, no. 12, pp. 1825-1835, 1999.

8 G. Oliveri, M. Donelli, and A. Massa, "Linear array thinning exploiting almost difference sets," *IEEE Trans. Antennas Propag.* (in press)

9 ELEDIA Almost Difference Set Repository (*http://www.eledia.ing.unitn.it/*)

10 C. Ding, T. Helleseth, and K. Y. Lam, "Several classes of binary sequences with three-level autocorrelation," *IEEE Trans. Inf. Theory*, vol. 45, no. 7, pp. 2606-2612, 1999.

11 K. T. Arasu, C. Ding, T. Helleseth, P. V. Kumar, and H. M. Martinsen, "Almost difference sets and their sequences with optimal autocorrelation," *IEEE Trans. Inf. Theory*, vol. 47, no. 7, pp. 2934-2943, 2001.