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# A Hybrid Approach to the Synthesis of Sub-arrayed Monopulse Linear Arrays

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**Abstract**

In this letter, a hybrid approach for the synthesis of the "optimal" compromise between sum and difference patterns for sub-arrayed monopulse antennas is presented. Firstly, the sub-array configuration is determined by exploiting the knowledge of the optimum difference mode coefficients to reduce the dimension of the searching space. In the second step, the sub-array weights are computed by means of a convex programming procedure, which takes advantages from the convexity, for a fixed clustering, of the problem at hand. A set of representative results are reported to assess the effectiveness of the proposed approach. Comparisons with state-of-the-art techniques are also presented.

**Key words**: Sum and difference patterns synthesis, contiguous partition, convex programming, hybrid optimization.

# 1 Introduction

In the recent literature, the use of a hybrid approach, namely, the *Simulated Annealing Convex Programming* ($Hybrid - SA$) method [1], for the synthesis of sub-arrayed monopulse linear antennas has improved the performances in shaping compromise patterns with respect to reference approaches [2]-[4]. By considering a sub-arraying strategy [5], the procedure proposed in [1] is aimed at finding "*the sub-array configuration and the coefficients of the sub-array sum signals such that the corresponding radiation pattern has a null with the maximum possible slope in a given direction, while being bounded by an arbitrary function elsewhere.*" Such a solution allows one the use of simpler feeding networks that guarantee both a reduced circuit complexity and low electromagnetic interferences as well as to obtain patterns with user-defined characteristics. It is based on the exploitation of the convexity of the functional with respect to a subset of the unknowns (i.e., the sub-array gains) and it is carried out by means of a *Convex Programming* ($CP$) method [1]. However, since the sub-array memberships of the array elements are determined by means of a Simulated Annealing ($SA$) algorithm, the procedure involves non-negligible computational costs to achieve the global minimum or there is the possibility that the solution is trapped in a local minimum (whether the criterion for the $SA$ convergence has not been verified [6]). In order to save computational resources, an innovative approach has been presented in [7]. It is an optimal pattern matching technique, namely the *Contiguous Partition Method* ($CPM$) [8], which has been integrated in an iterative procedure considering different reference patterns to deal with constraints on the level of the sidelobes ($SLL$), as well. The $CPM$ takes advantage from the knowledge of the optimal excitations of the difference pattern [9][10][11] and from the concept of contiguous partitions [12] to reduce the searching space and, thus, effectively handling the problem of the optimal clustering. As a matter of fact, the arising computational burden turns out to be significantly reduced compared to that of previous optimization schemes.

In this letter, a hybrid approach (called $Hybrid - CPM$ method), which integrates the $CPM$ [8] with a gradient-based $CP$ procedure [1] to profitably benefit of the positive features of both $CPM$ and $CP$ approach is carefully described and validated. At the first step, the "optimal" sub-array configuration is computed according to the procedure described in [8] by exploiting the relationship between the excitation coefficients of the optimal sum [14][15][16][17] and difference [9][10][11] modes. Once the clustering has been determined, the sub-array gains are computed as in [1].

## 2 Mathematical Formulation

Let us consider a linear array of $N = 2M$ equally-spaced isotropic elements $a_n$, $n = -M, \dots, -1, 1, \dots, M$ and the corresponding space factor given by:

$$f(\theta) = \sum_{n=-M}^{M} a_n e^{j(n - sgn(n)/2)kd\cos(\theta)} \tag{1}$$

where $k$ and $d = \frac{\lambda}{2}$ are the wavenumber of the background medium and the inter-element spacing, respectively. Moreover, $\theta$ indicates the angular rotation with respect to the direction orthogonal to the array. It is well known that optimal sum [14][15][16][17] and difference [9][10][11] patterns are afforded by independent sets of symmetric $\underline{A}^s = \{a_n^s;\, n = \pm1, ..., \pm M\}$ and anti-symmetric $\underline{A}^d = \left\{a_n^d;\, n = \pm1, ..., \pm M\right\}$ excitations, therefore the corresponding array space factors (1) turns out to be even $[f^s(\theta) = f^s(-\theta)]$ and odd $[f^d(\theta) = -f^d(-\theta)]$ functions [1]. Consequently, only half of the array elements are descriptive of the whole array. In order to yield at the same time optimal sum and difference patterns, two independent and complete feeding networks are usually needed. However, such a solution is generally very expensive and impractical due to the circuit complexity, the physical space limitations, and the electromagnetic interferences. Therefore, the sub-arraying strategy is usually adopted since it allows a suitable trade-off between the antenna feasibility and the synthesized pattern features.

The $Hybrid-CPM$ approach belongs to sub-arraying techniques, but unlike the $Hybrid-SA$, it considers a two-stage-iterative procedure instead of an iterative one step process wherein each step involves in turn the solution of a convex optimization problem. The first step is based on the $CPM$ (i.e., a matching method likewise the Excitation Matching Method ($EMM$) proposed by McNamara in [5]) and it is aimed at defining the sub-array configuration $\underline{C}^{CPM}$ that minimizes the following cost function

$$\Psi^{CPM}\left(\underline{C}\right)=\sum_{m=1}^{M}\left|a_m^s\left\{\frac{a_m^d}{a_m^s}-\sum_{q=1}^{Q}\delta_{c_mq}\left[\frac{\sum_{j=1}^{M}\delta_{qc_j}\left(a_j^sa_j^d\right)}{\sum_{j=1}^{M}\delta_{qc_j}\left(a_j^s\right)^2}\right]\right\}\right|^2 \tag{2}$$

obtained after simple algebra from the functional used in [5] and aimed at quantifying the distance in the mean square norm of the synthesized solution to the independently reference difference set $\underline{A}^d$. In Eq. (2), $\underline{C}=\{c_m;\, m=1,\ldots,M\}$ is a vector of integer values (i.e., $c_m\in[1,Q]$) that identifies the sub-array membership of each element of the array [4], $q$ is the sub-array index and $\delta_{qc_m}$ is the Kronecker delta (i.e., $\delta_{qc_m}=1$ if $q=c_m$, $\delta_{qc_m}=0$ otherwise). The solution of such a problem is "*a contiguous partition of $M$ completely ordered elements into $Q$ subsets that may be represented by $Q-1$ points of division lying in any of the $M-1$ intervals between adjacent elements*" [12]. This solution represents the best step-wise approximation of the considered partition and "*the number of possible contiguous partitions is equal to the number of ways of choosing the division points, which is the number of combinations of $M-1$ different things taken $Q-1$ at a time* [i.e., $U^{CPM}=\begin{pmatrix}M-1\\Q-1\end{pmatrix}$, $U^{CPM}$ being the number of contiguous partition]". Accordingly, $\underline{C}^{CPM}$ is determined by generating a sequence of contiguous partitions $\left\{\underline{C}^{(k)};\, k=0,...,K\right\}$ starting from a guess aggregation $\underline{C}^{(0)}$ and updating the solution [$\underline{C}^{(k)}\leftarrow\underline{C}^{(k+1)}$] just modifying the membership of the "*border elements*" [7] of the array by means of the local search strategy presented in [7].

The second step exploits the following property [1]: "*the optimal compromise between sum and difference patterns is a convex problem with respect to the sub-array weights for a fixed sub-array configuration $\underline{C}$*". Accordingly, once the element membership has been determined [i.e., $\underline{C}^{(opt)}=\underline{C}^{CPM}$], the optimal weight vector $\underline{W}^{(opt)}$ is computed by minimizing the following cost function

$$\Psi^{CP}\left(\underline{W}\right)=\left.\frac{d\Re\left\{f^d\left(\theta\right)\right\}}{d\theta}\right|_{\theta=\theta_0} \tag{3}$$

subject to $\left.\frac{d\Im\{f^d(\theta)\}}{d\theta}\right|_{\theta=\theta_0} = 0$ and $\left|f^d(\theta)\right|^2 \leq \aleph(\theta)$, where $\theta_0$ indicates the boresight direction and $\aleph(\theta)$ is a non-negative function that defines the upper bounds for the sidelobes. Moreover, $\underline{W} = \{w_q;\, q = 1, \ldots, Q\}$ is the sub-array weight vector and $\Re$ and $\Im$ denote the real part and the imaginary one, respectively. Towards this end, a standard gradient-based optimization is performed by generating a succession of trial solutions $\left\{\underline{W}^{(h)};\, h = 0, ..., H\right\}$ starting from the initial guess given by $\underline{W}^{(0)} = \left\{w_q^{CPM};\, q = 1, \ldots, Q\right\}$ being $w_q^{CPM} = \left[\frac{\sum_{j=1}^{M} \delta_{qc_j}\left(a_j^s a_j^d\right)}{\sum_{j=1}^{M} \delta_{qc_j}\left(a_j^s\right)^2}\right]$.

# 3 Numerical Assessment

In this section, the effectiveness and potentialities of the proposed hybrid method will be assessed dealing with three benchmarks of the related literature in order to complete the preliminary validation presented in [13] and to further confirm, in a more exhaustive fashion, the underlying proof-of-concept. As a matter of fact, the test cases under analysis are concerned with linear arrays and, for the sake of completeness, with both a small ($M = 10$) and a large ($M = 100$) number of elements. Whatever the experiment, the synthesis is aimed at minimizing the $SLL$ of the compromise difference pattern for a fixed beamwidth or, analogously, at maximizing the slope along the boresight direction [1] fixed at $\theta_0 = 0^o$.

The first test case deals with a linear array of $N = 20$ elements. As far as the sum mode is concerned, it has been fixed to a Villeneuve sum pattern [16], with $\bar{n} = 4$ and $SLL = -25\,dB$, in the first experiment, whereas a Dolph-Chebyshev [14] pattern with $SLL = -20\,dB$ has been chosen for the second one. In the first experiment, a configuration with $Q = 5$ sub-arrays and uniform clustering is considered. Moreover, as regards the optimal/reference difference pattern of the approaches that exploit the concept of contiguous partitions, the excitations $\underline{A}^d$ have been fixed to a modified Zolotarev distribution ($\overline{n} = 4$, $\varepsilon = 3$) whose pattern is characterized by $SLL_{ref} = -25\,dB$. Figure 1 pictorially compares the patterns obtained with the $EMM$ [5], the $CMP$ [8], and the $Hybrid{-}CPM$ approach, whose final sub-array configuration and weights are $\underline{C}^{(opt)} = \{1\,1\,2\,3\,3\,5\,5\,4\,4\,2\}$

and $\underline{W}^{(opt)} = \{0.3352, 1.1299, 1.3708, 1.8309, 1.8699\}$, respectively. It is worth noting that the $Hybrid-CPM$ approach outperforms other methods with a reduction of over $5\,dB$ and more than $1\,dB$ of the the $SLL$ with respect to the $EMM$ and the $CPM$, respectively (Tab. I).

The second experiment is devoted to complete the comparison by considering the state-of-the-art methods based on stochastic optimizations. In particular, the results from the $Hybrid-SA$ [1] and the Differential Evolution ($DE$) optimization algorithm [4] have been taken into account. The array configuration is that with $Q=8$. The array patterns obtained from the application of the $CPM$-based methods according to the guidelines in [8] and by assuming a reference Zolotarev pattern [10] with $SLL_{ref}=-39\,dB$ are shown in Fig. 2$(a)$ together with those from the other approaches. With reference to Fig. 2$(a)$ and as quantitatively estimated in Tab. I, the $Hybrid-CPM$ plot presents a $SLL$ of $-37.5\,dB$ (i.e., almost $1\,dB$ below the $SLL$ of the $Hybrid-SA$ [1] and more than $15\,dB$ when compared to the pattern in [4] with the same number of sub-arrays), with $\underline{C}^{(opt)} = \{2\,3\,5\,7\,8\,8\,6\,4\,3\,1\}$ and $\underline{W}^{(opt)} = \{1.1836, 1.8818, 4.9795, 6.9286, 7.3462, 8.5109, 9.1480, 9.7003\}$. Furthermore, it is worth analyzing the beamwidths ($BW$s) (or, similarly, the first null positions) of the results in Fig. 2$(a)$. As a matter of fact, the $Hybrid-CPM$ solution presents not only the lowest $SLL$ value, but also the narrower $BW$ (i.e., $BW_{Hybrid-CPM}=0.097$ vs. $BW_{Hybrid-SA}=0.102$ and $BW_{DE}=0.113$). Such a result further confirms the effectiveness of the $Hybrid-CPM$ in dealing with the non-convex part of the problem at hand, thus allowing the synthesis of compromise patterns with better characteristics. As expected, the improvements in terms of $SLL$ are even larger by setting the same $BW$ constraint used with $Hybrid-SA$ [1]. Towards this aim, the reference excitations $\underline{A}^d$ have been chosen to afford a Zolotarev difference pattern [10] with $SLL_{ref}=-41\,dB$. In such a case, the achieved solution has a $SLL=-38.0\,dB$ with an improvement of about $0.5\,dB$ [Tab. I] compared to that in Fig. 2$(a)$. For completeness, the values of the obtained clustering and sub-array weights are equal to $\underline{C}^{(opt)} = \{2\,4\,6\,8\,8\,8\,7\,5\,3\,1\}$ and $\underline{W}^{(opt)} = \{0.7461, 2.0518, 4.0934, 5.4616, 6.5563, 8.2545, 8.5060, 10.0768\}$, respectively.

As far as the computational costs are concerned, the number of iterations, $K$, required to get the final clustering starting from a uniform one at the initialization, is $K_{CPM} = 4$ and $K_{CPM} = 3$, for the two $CPM$-based syntheses, respectively, and the total $CPU$-time is shorter than 10 $[\mu sec]$ in both cases. Moreover, the whole synthesis time of the $Hybrid - CPM$ amounts to 3.078 $[sec]$ and 3.781 $[sec]$, respectively. As regards to the higher burden of the $Hybrid - CPM$ compared to the $CPM$, this is due to the solution of the $CP$ problem, which ends in $K_{CP} = 18$ iterations. For comparative purposes, let us notice that a greater computational burden affects the $Hybrid - SA$ [1] method since $K_{Hybrid-SA} = 25$ have been chosen and $CP$ problem is solved at each iteration. Similar conclusions hold true also for the $DE$ approach [4] where the number of iterations has been set to $K_{DE} = 10$.

The last comparative example deals with the synthesis of a large array ($N = 200$). Thanks to the computational saving [18], the $CPM$-based procedures are able to effectively face with such a problem dimensionality. The sum coefficients have been chosen to generate a Dolph-Chebyshev [14] pattern with $SLL = -25\,dB$, while the values of the reference difference excitations have been fixed to those of the Zolotarev difference pattern with $SLL_{ref} = -30\,dB$. The behaviors of the patterns in Fig. 3 clearly point out that the integration of the $CP$ optimization with the $CPM$ allows a non-negligible enhancement of the $SLL$ performances. As a matter of fact, the $SLL$ computed in correspondence with the clustering determined by the $Hybrid-CPM$ method (Tab. II) is of about $3\,dB$ lower than that of the standard version of the $CPM$ (see Tab. I).

Finally, in order to assess the reliability of the synthesized solutions, let us evaluate the radiated power patterns when mutual coupling ($MC$) effects are included into the array model. Towards this purpose, the $MC$ models proposed in [19] and [20] have been taken into account and compared as in [21]. The case-of-study example deals with a 20-element uniform linear array of thin $\lambda/2$ dipoles oriented along the $z$ axis [22]. As a representative example, the effects of the $MC$ on the solution obtained with the $Hybrid-CPM$ approach and shown in Fig. 1 are analyzed. Figure 4 shows the pictorial representations of the relative power patterns for different situations. As it can be observed, the radiation pattern obtained by including the $MC$ effects is similar to the ideal case whatever the considered $MC$ model. More in detail, the null positions are equal to those of the ideal pattern, while some perturbations only affect the behavior of the secondary lobes without compromising the performance of the difference beam.

# 4 Conclusions and Discussions

In this letter, a hybrid approach devoted to the synthesis of the "optimal" compromise between sum and difference patterns for sub-arrayed monopulse antennas has been presented. In such a method, the element memberships are defined through the $CPM$ that exploits the knowledge of the optimal difference mode coefficients to reduce the set of admissible sub-array configurations and to speed up the convergence of the compromise synthesis. The sub-array gains are then computed by means of a convex programming procedure that takes advantage from the convexity of the arising cost function in correspondence with a fixed clustering. Representative results have been reported in order to assess the potentialities of the proposed $Hybrid - CPM$ technique in dealing with the synthesis of both small and large monopulse arrays, where mutual coupling effects have been taken into account, as well.

Concerning the optimization problem at hand, the proposed $CPM$-based procedure does not guarantee that the retrieved sub-array configuration is the best choice for optimizing the $SLL$. As a matter of fact, such a configuration can be (theoretically) obtained only by means of global optimization procedures. However, the proposed procedure has shown to outperform state-of-the-art global optimization strategies. Furthermore, starting from the assumption that $CPM$-based strategies are matching techniques, the proposed approach can be easily extended to arbitrary sidelobe masks or pattern shapes (for both sum and difference patterns) by profitably using the state-of-the-art approaches (e.g., [17][11]) to set the reference patterns. Future research works will be aimed at implementing such extensions and different antenna applications.

## Acknowledgment

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# FIGURE CAPTIONS

- **Figure 1.** *Uniform Sub-arraying* ($M = 10$, $Q = 5$) - Normalized compromise difference patterns obtained by means of the $Hybrid - CPM$ method, the $CPM$ [8], and the $EMM$ [5].

- **Figure 2.** *Non-Uniform Sub-arraying* ($M = 10$, $Q = 8$) - Normalized compromise difference patterns obtained by means of the $Hybrid - CPM$ method, the $CPM$ [8], the $SA - CP$ approach [1], and the $DE$ optimization [4].

- **Figure 3.** *Large Arrays* ($M = 100$, $Q = 6$) - Normalized compromise difference patterns obtained with the $Hybrid - CPM$ method and the $CPM$ [8].

- **Figure 4.** *Mutual Coupling* ($M = 10$, $Q = 5$) - Normalized compromise difference patterns obtained with the $Hybrid - CPM$ in correspondence with ideal sources and dipoles without and with mutual coupling effects.

# TABLE CAPTIONS

- **Table I.** Values of the $SLL$ of the array factors in Figs. 1-3.

- **Table II.** *Large Arrays* ($M = 100$, $Q = 6$) - Sub-array configuration and weights determined by the $Hybrid - CPM$ method (see Fig. 3 for the corresponding pattern).

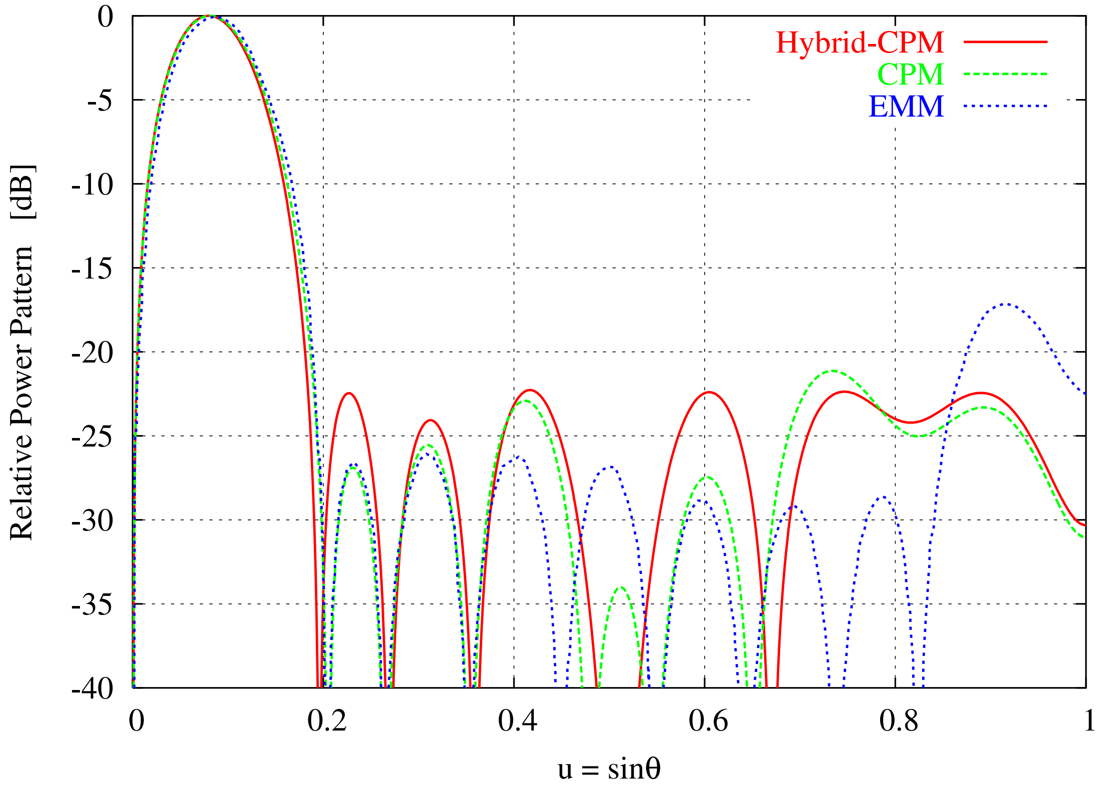
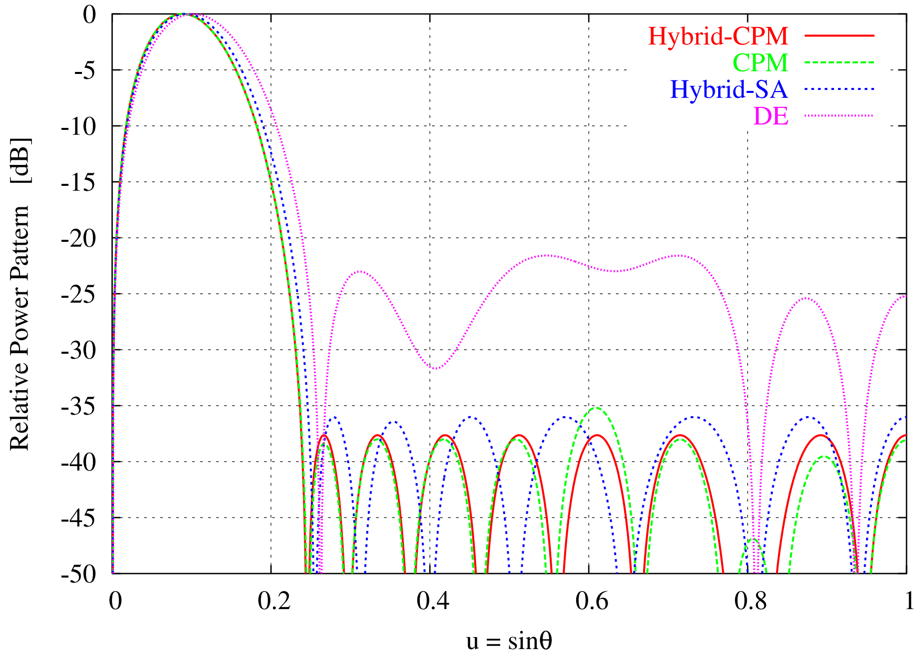
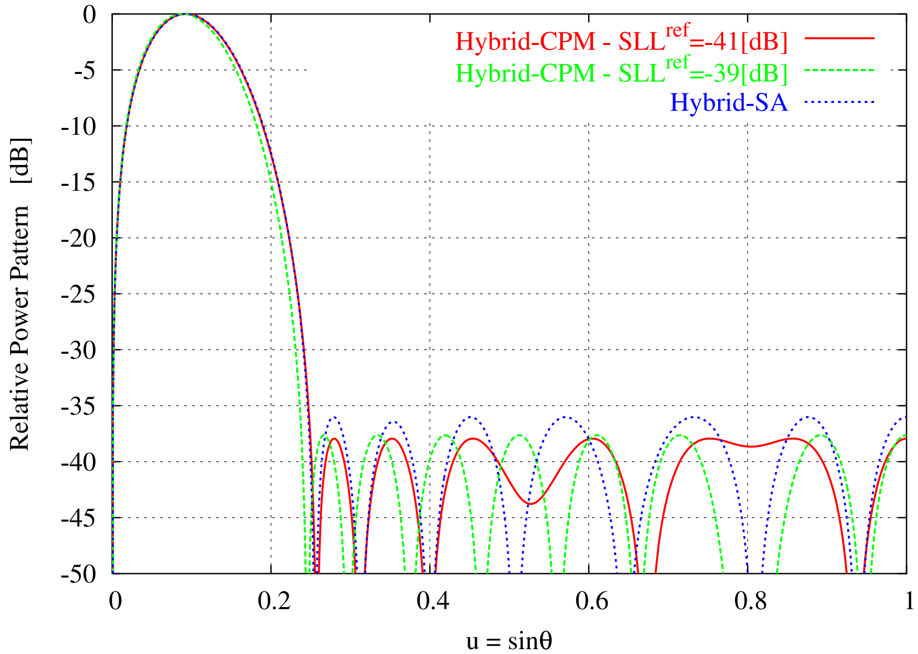



**Fig. 1 - P. Rocca *et al.*,** "A Hybrid Approach for the Synthesis ..."




(*a*)




(*b*)

**Fig. 2 - P. Rocca *et al.*,** "A Hybrid Approach for the Synthesis ..."

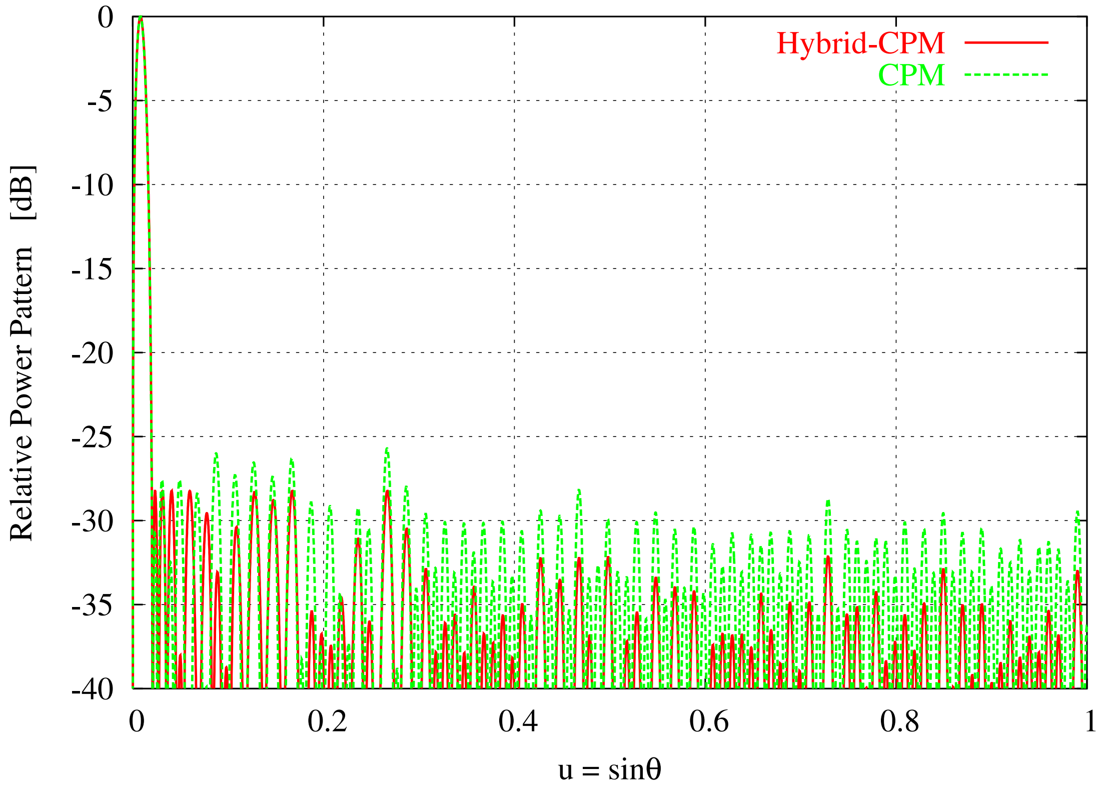



**Fig. 3 - P. Rocca *et al.*,** "A Hybrid Approach for the Synthesis ..."

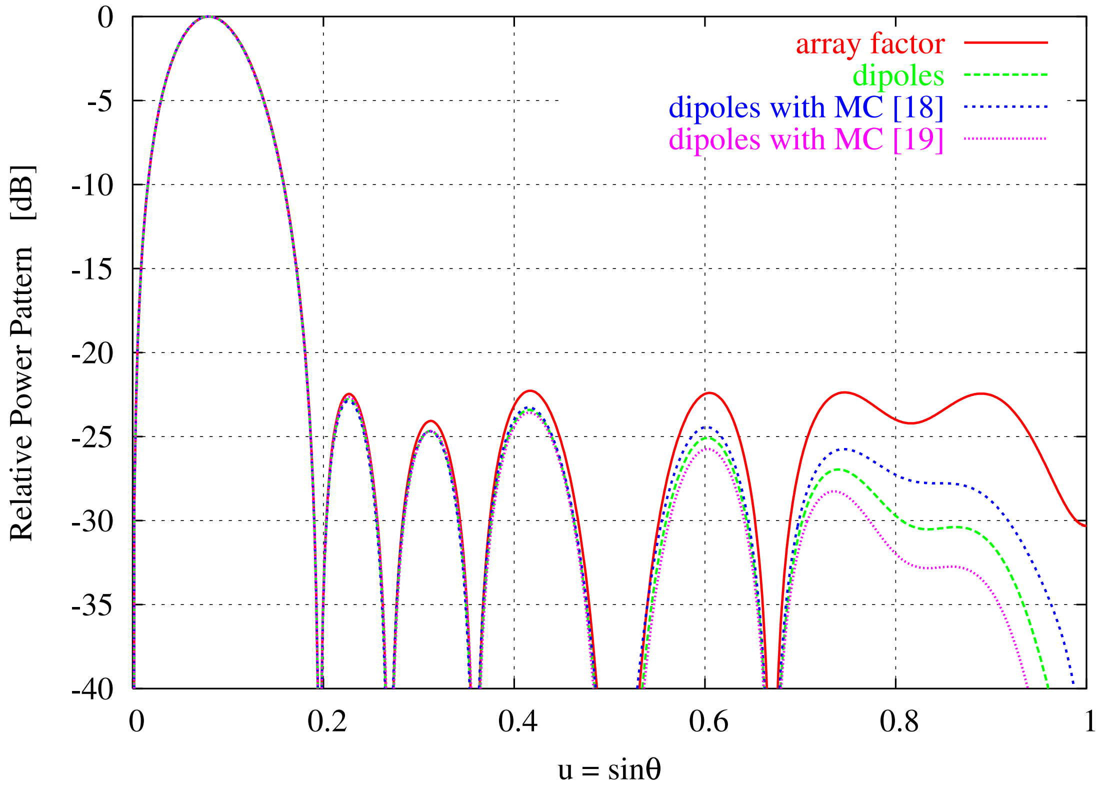



**Fig. 4 - P. Rocca** ***et al.*****,** "A Hybrid Approach for the Synthesis ..."

**Tab. I - P. Rocca *et al.*,** "A Hybrid Approach for the Synthesis ..."

| $[dB]$ | | $Reference$ | $Hybrid - CPM$ | $CPM$ | $EMM$ | $Hybrid - SA$ | $DE$ |
|---|---|---|---|---|---|---|---|
| $M = 10$ | $Q = 5$ | $-25.0$ | $-22.4$ | $-21.0$ | $-17.0$ | $-$ | $-$ |
| $M = 10$ | $Q = 8$ | $-39.0$ | $-37.5$ | $-35.2$ | $-$ | $-36.5$ | $-21.6$ |
| $M = 10$ | $Q = 8$ | $-41.0$ | $-38.0$ | $-32.7$ | $-$ | $-36.5$ | $-21.6$ |
| $M = 100$ | $Q = 6$ | $-30.0$ | $-28.3$ | $-25.7$ | $-$ | $-$ | $-$ |

<table>
<tr><td>$M = 100$</td><td>$\underline{C}$</td><td colspan="6">11111111111111222222223333333344444444555555555666<br>66666666666666666666666666666555555555444444433331</td></tr>
<tr><td>$Q = 6$</td><td>$\underline{W}$</td><td>0.2133</td><td>0.7235</td><td>0.9417</td><td>1.0909</td><td>1.2752</td><td>1.4294</td></tr>
</table>

**Tab. II - P. Rocca *et al.*,** "A Hybrid Approach for the Synthesis ..."