Statistical Analysis of an Inverse Scattering BCS-based Technique under the Born Approximation for the Reconstruction of Sparse Scatterers

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Abstract

In this report, a statistical analysis of the BCS-based technique aimed to reconstruct sparse lossless cylinders has been reported. Different investigation scenarios with multiple small cylinders located in different positions have been proposed, considering also different dielectric features of the objects.

Contents

1 TEST CASE: Statistical Analysis

Goal: evaluate the performances of BCS

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $N = 324$

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

Object:

- Square cylinder of side $\frac{\lambda}{6} = 0.16667$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$ [S/m]

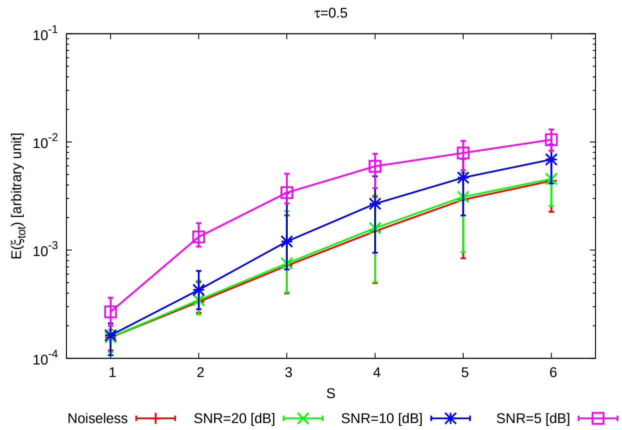
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergenze parameter: $\tau = 1.0 \times 10^{-8}$

Statistical Analysis:

- $K = 10$ random seeds used for each case

RESULTS: $\varepsilon_r = 1.5$



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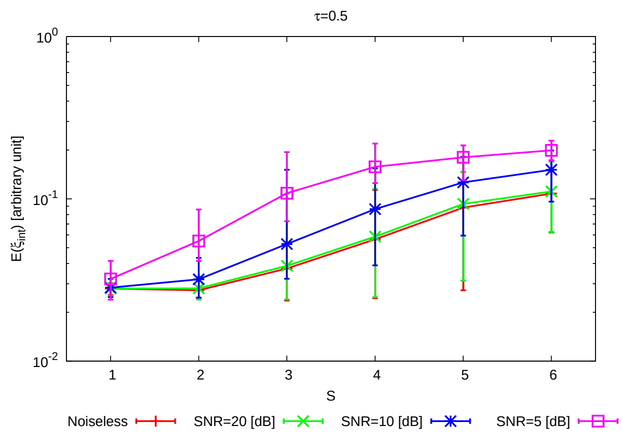

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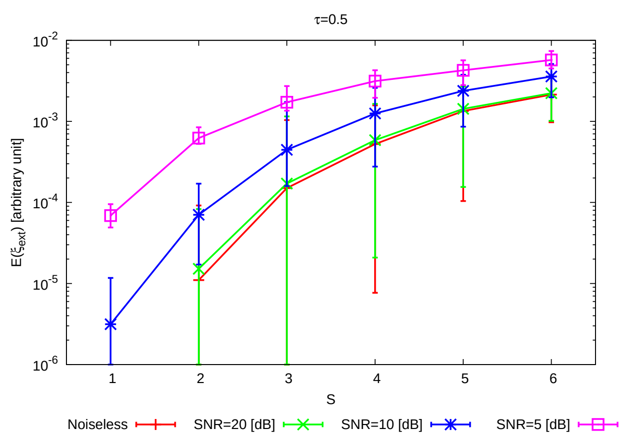
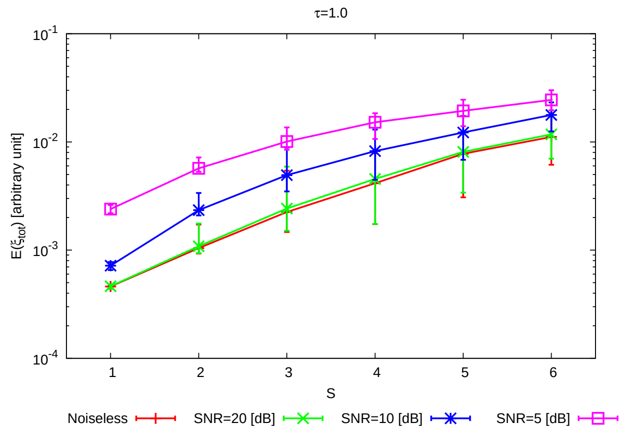


(c)

Figure 147. *Statistical analysis* $[K = 10,\ \varepsilon_r = 1.5]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor), for different SNR values: (a) total error ξ_{tot}, (b) internal error ξ_{int}, (c) external error ξ_{ext}.

RESULTS: $\varepsilon_r = 2.0$



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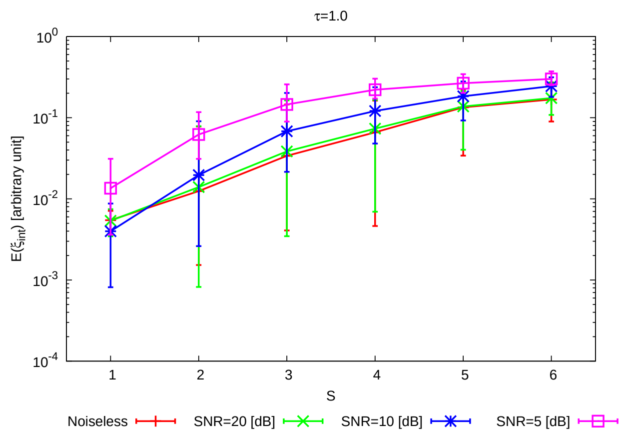

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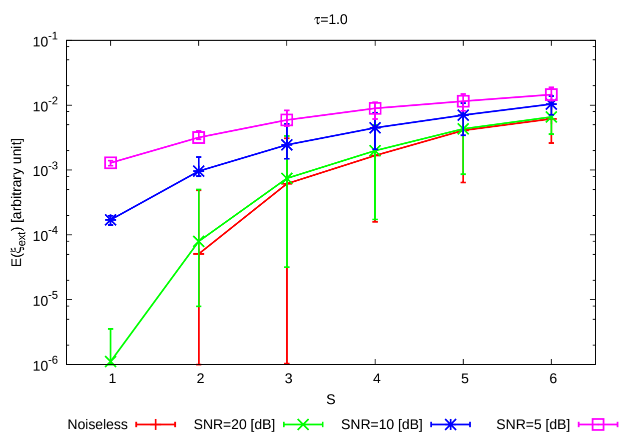
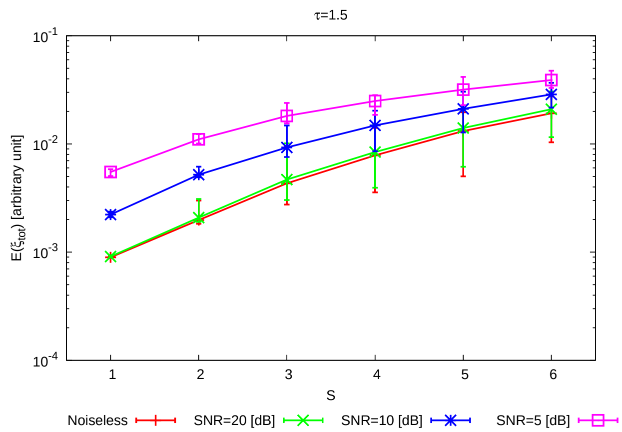


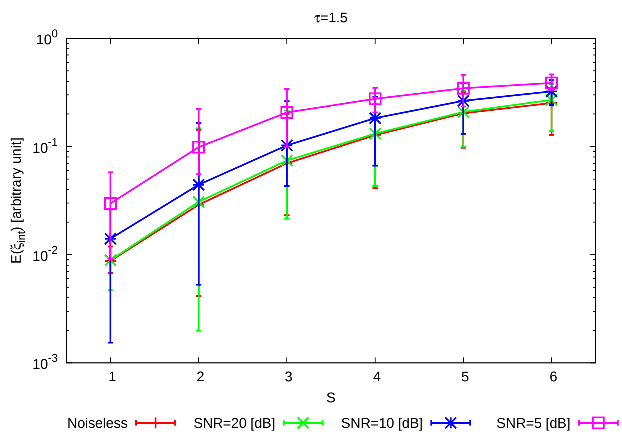
(c)

Figure 148. *Statistical analysis* $[K = 10, \varepsilon_r = 2.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor), for different SNR values: (a) total error ξ_{tot}, (b) internal error ξ_{int}, (c) external error ξ_{ext}.

RESULTS: $\varepsilon_r = 2.5$



(a)



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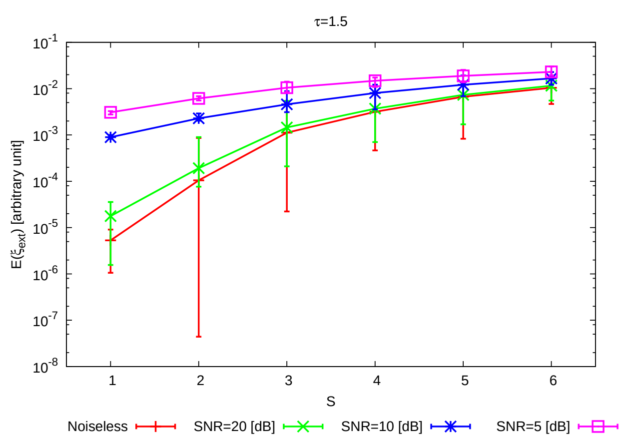
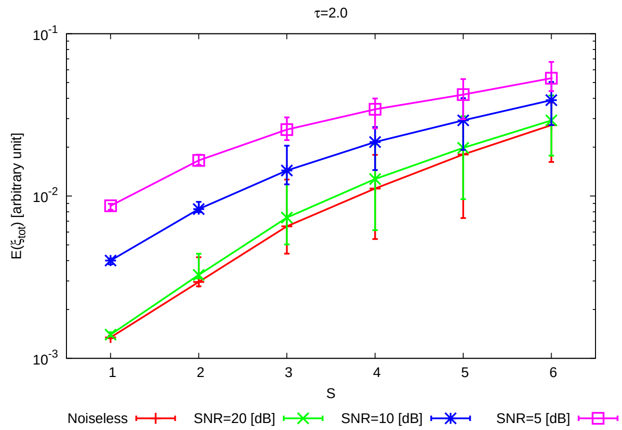


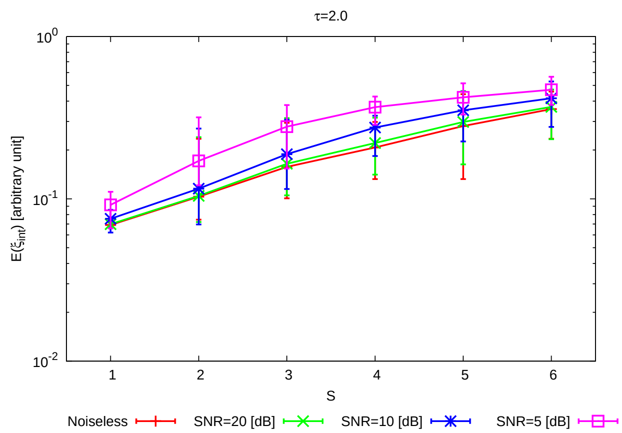
(c)

Figure 149. *Statistical analysis* $[K = 10,\ \varepsilon_r = 2.5]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor), for different SNR values: (a) total error ξ_{tot}, (b) internal error ξ_{int}, (c) external error ξ_{ext}.

RESULTS: $\varepsilon_r = 3.0$



(a)



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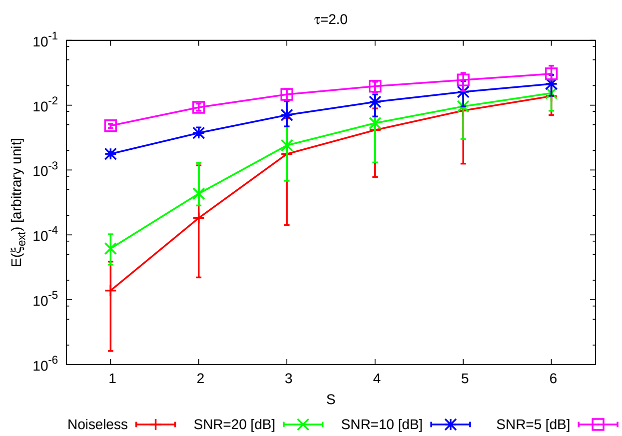

(c)

Figure 150. *Statistical analysis* $[K = 10, \varepsilon_r = 3.0]$ - Behaviour of mean, maximum and minimum of the error figures as a function of S (sparsity factor), for different SNR values: (a) total error ξ_{tot}, (b) internal error ξ_{int}, (c) external error ξ_{ext}.

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