

Reconstruction of Sparse Scatterers through an Approximated Compressive Sensing Strategy

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Abstract

This report proposes a numerical validation of the Bayesian Compressive Sampling-based technique for imaging dielectric cylinders within the conditions of "sparsity" and "weakness" of the scatterers. Various shape of the scatterers and different values of dielectric permittivity have been considered.

Contents

1	TEST CASE: Two Square Cylinders	3
2	TEST CASE: Three Square Cylinders	9
3	TEST CASE: Four Square Cylinders	15
4	TEST CASE: Cross-Shaped Cylinder	21
5	TEST CASE: L-Shaped Cylinder	27
6	TEST CASE: Inhomogeneous L-Shaped Cylinder	33

1 TEST CASE: Two Square Cylinders

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

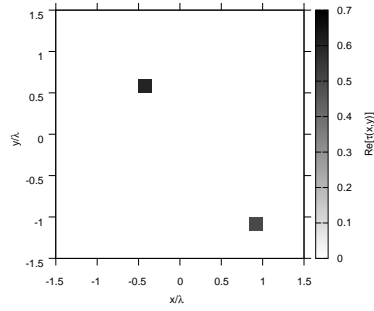
Object:

- Two square cylinders of side $\frac{\lambda}{6} = 0.1667$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$ (one square), $\varepsilon_r = 1.9$ (one square)
- $\sigma = 0$ [S/m]

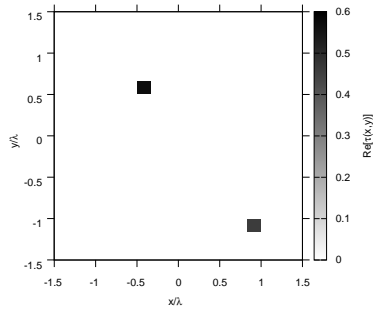
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

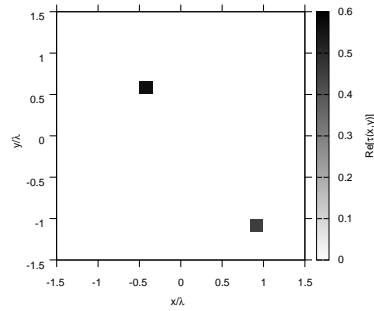
RESULTS: $\varepsilon_r = 1.5$



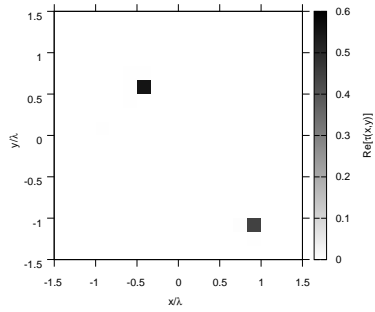
(a)



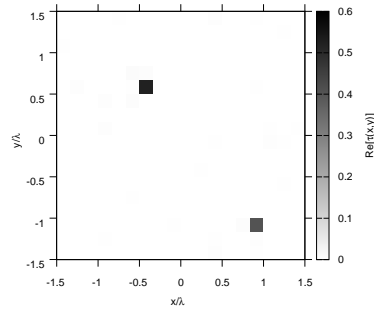
(b)



(c)



(d)



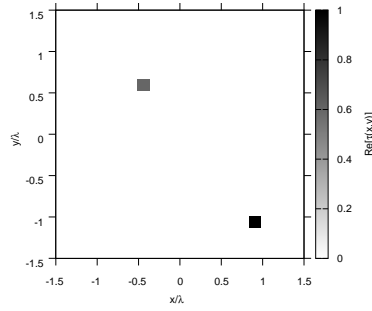
(e)

Figure 17. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

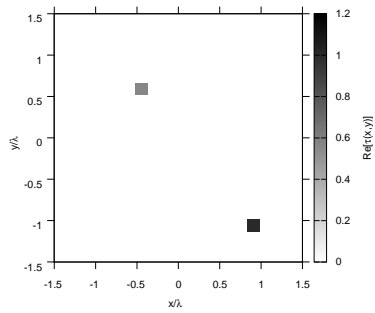
Observations:

Ricostruzioni molto buone per tutti i valori di SNR .

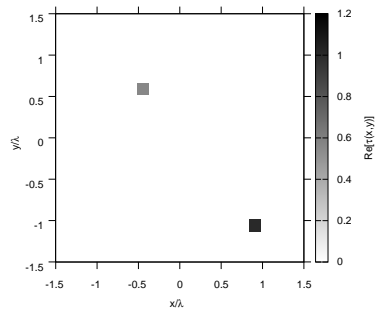
RESULTS: $\varepsilon_r = 2.0$



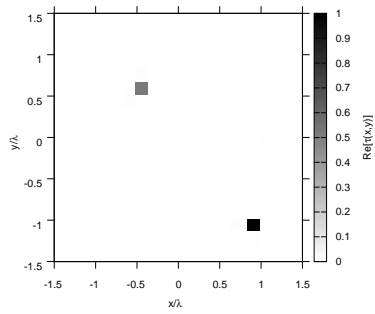
(a)



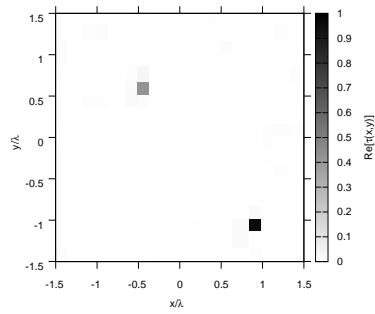
(b)



(c)



(d)



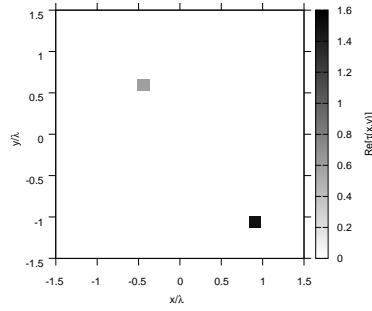
(e)

Figure 18. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

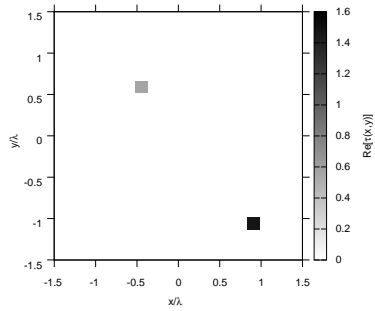
Observations:

Ricostruzioni in generale buone per tutti i valori di SNR .

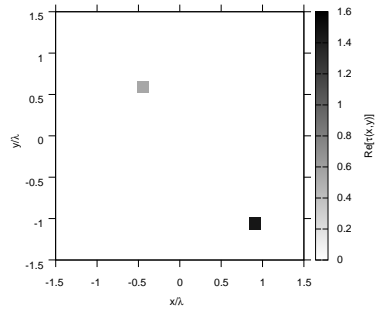
RESULTS: $\varepsilon_r = 2.5$



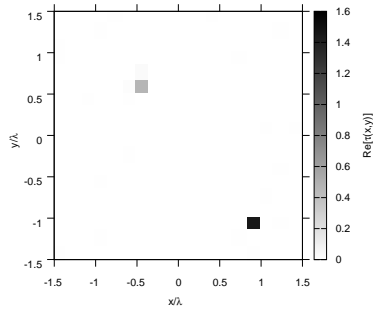
(a)



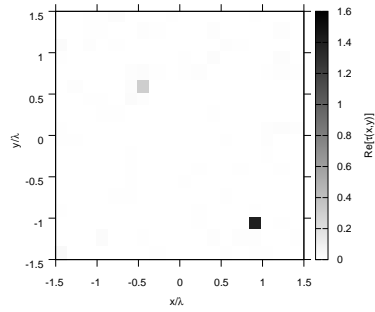
(b)



(c)



(d)



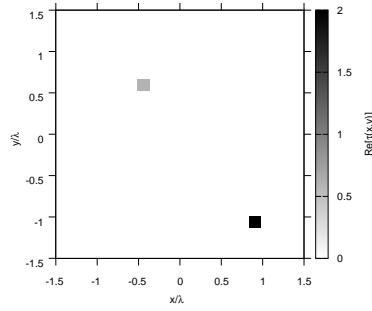
(e)

Figure 19. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

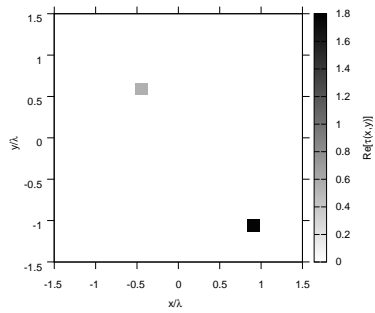
Observations:

Ricostruzioni in generale buone per tutti i valori di SNR .

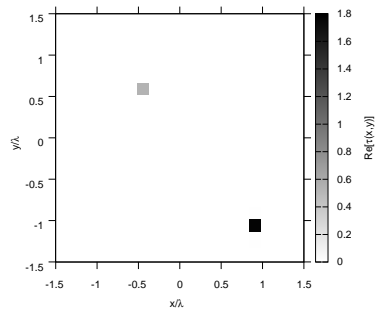
RESULTS: $\varepsilon_r = 3.0$



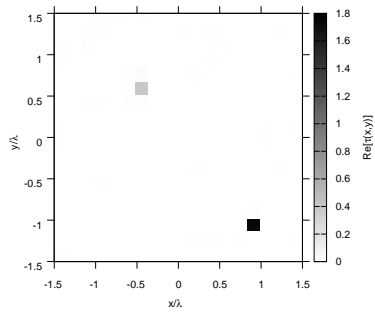
(a)



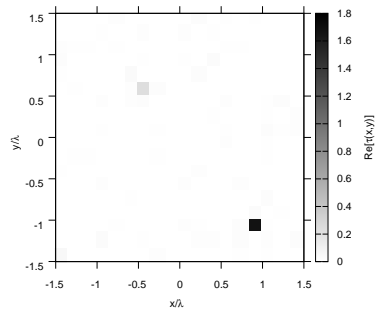
(b)



(c)



(d)



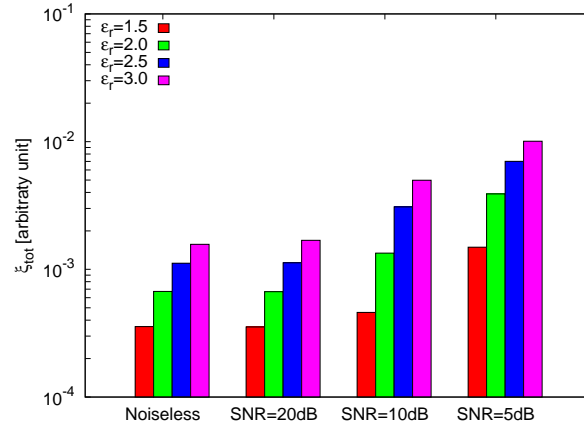
(e)

Figure 20. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

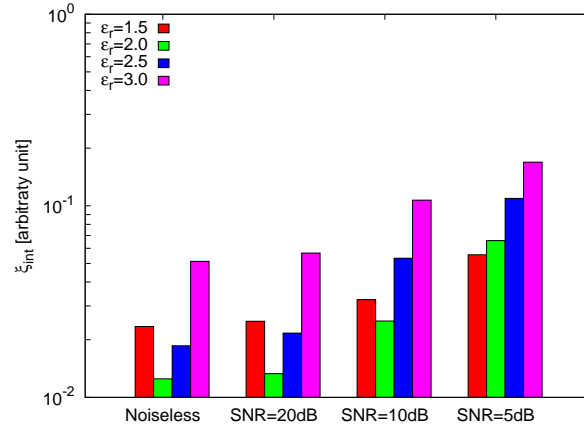
Observations:

Ricostruzioni in generale buone per tutti i valori di SNR .

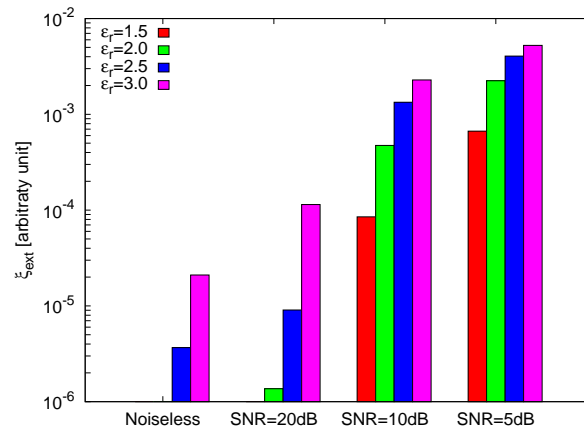
RESULTS: Error Figures



(a)



(b)



(c)

Figure 21. Behaviour of error figures as a function of ε_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

2 TEST CASE: Three Square Cylinders

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
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- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
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- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

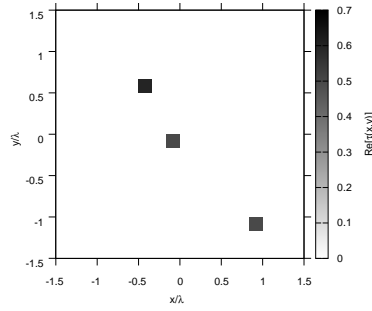
Object:

- Three square cylinders of side $\frac{\lambda}{6} = 0.1667$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$ (two squares), $\varepsilon_r = 1.9$ (one square)
- $\sigma = 0$ [S/m]

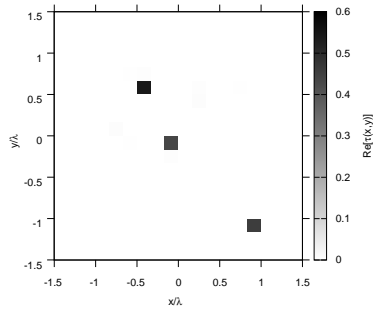
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

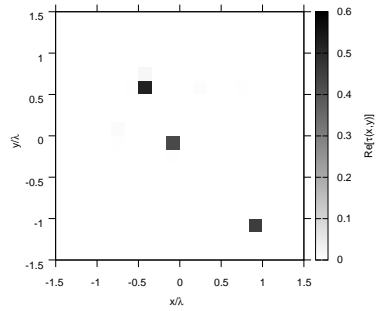
RESULTS: $\varepsilon_r = 1.5$



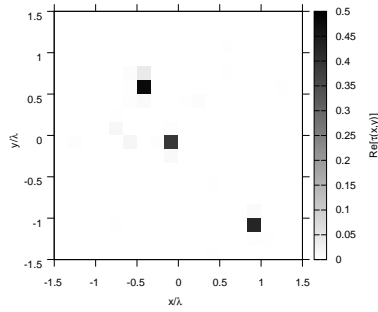
(a)



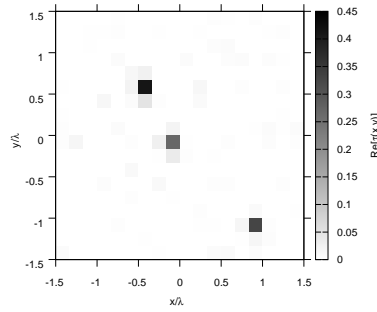
(b)



(c)



(d)



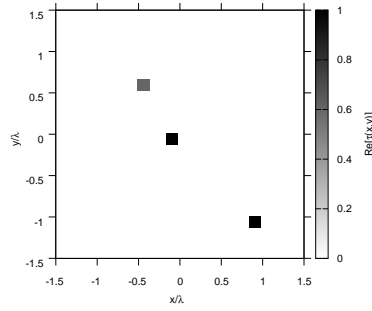
(e)

Figure 22. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB], (d) $SNR = 10$ [dB], (e) $SNR = 5$ [dB].

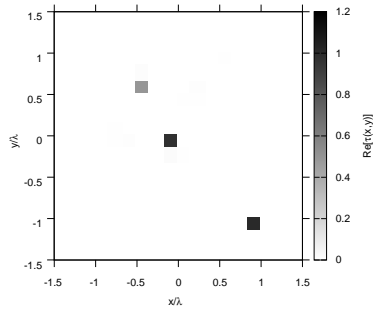
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

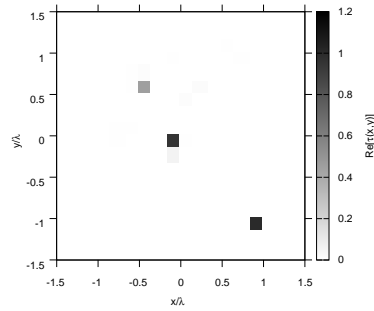
RESULTS: $\varepsilon_r = 2.0$



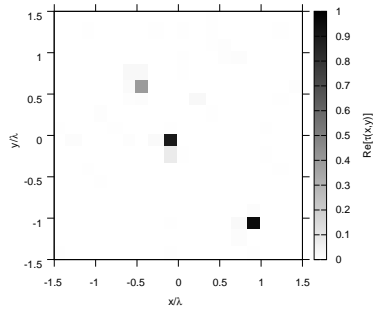
(a)



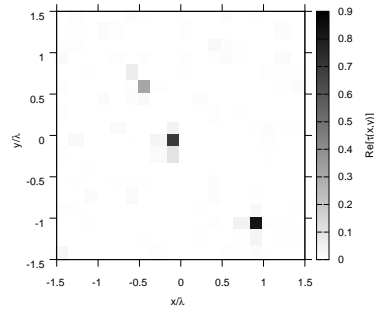
(b)



(c)



(d)



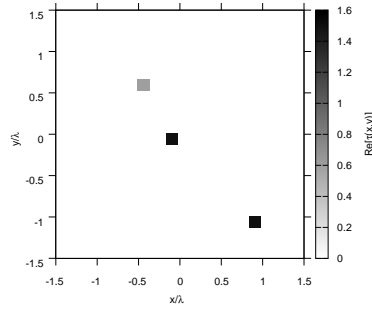
(e)

Figure 23. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

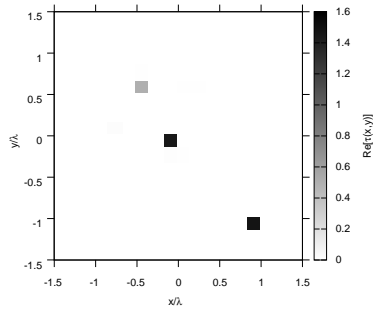
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

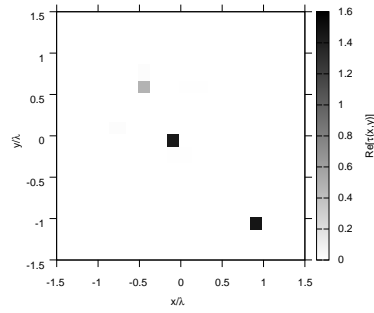
RESULTS: $\varepsilon_r = 2.5$



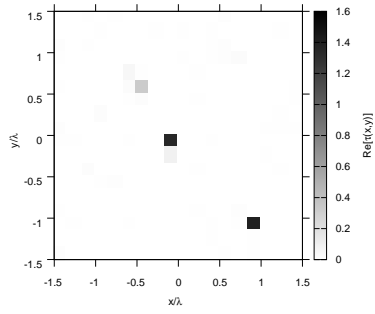
(a)



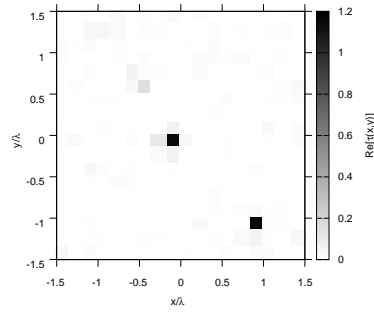
(b)



(c)



(d)



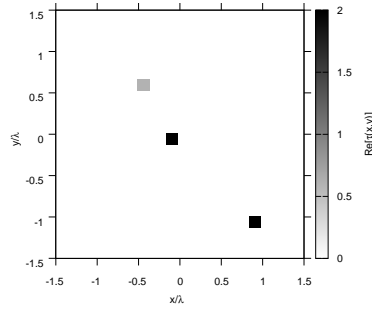
(e)

Figure 24. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

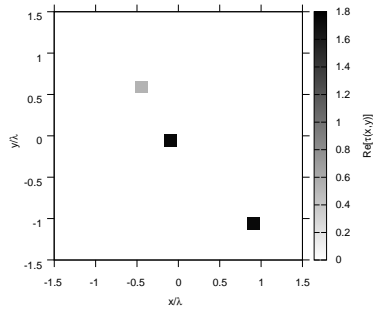
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

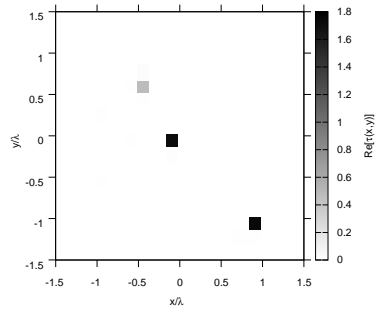
RESULTS: $\varepsilon_r = 3.0$



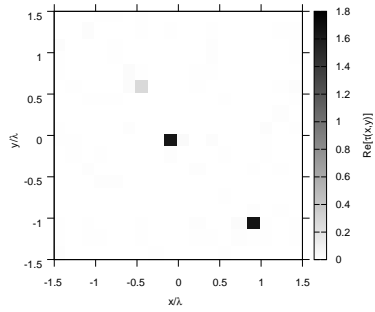
(a)



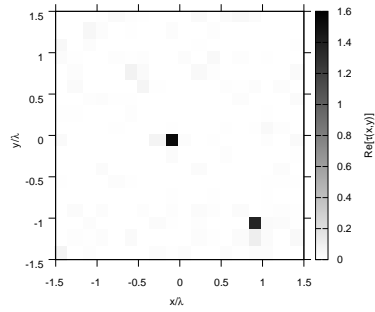
(b)



(c)



(d)



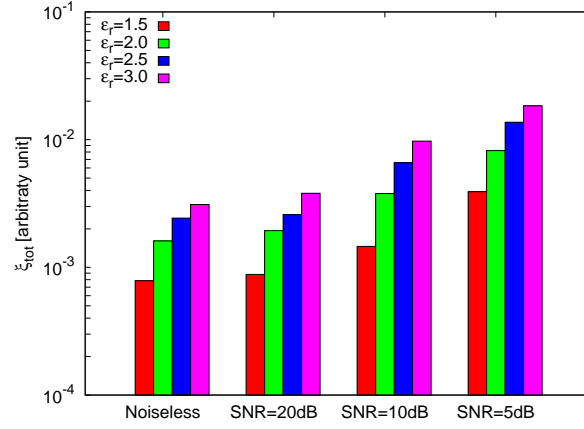
(e)

Figure 25. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

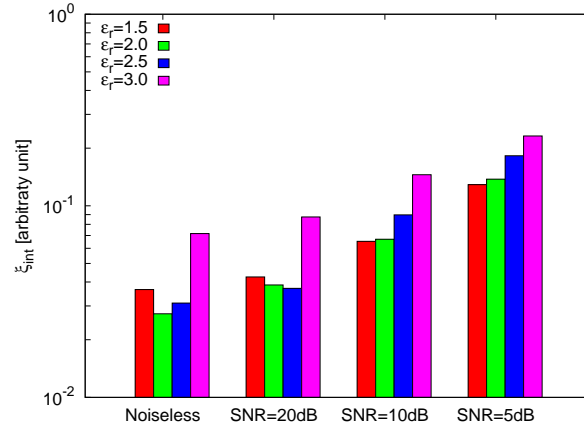
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

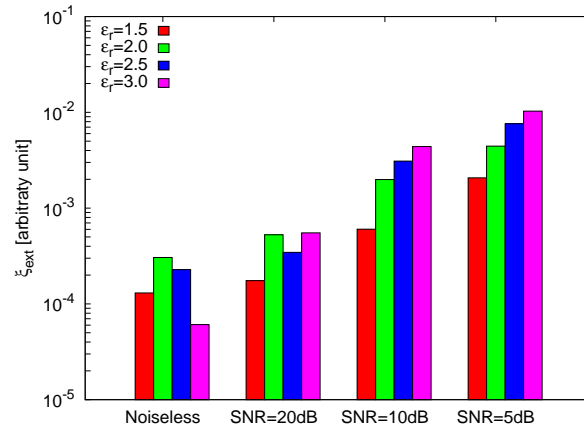
RESULTS: Error Figures



(a)



(b)



(c)

Figure 26. Behaviour of error figures as a function of ϵ_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

3 TEST CASE: Four Square Cylinders

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
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Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
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Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

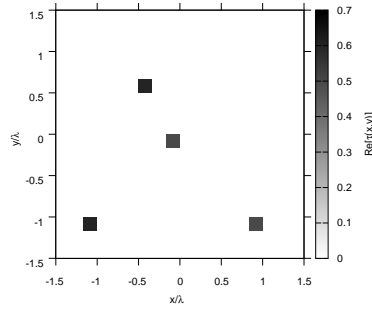
Object:

- Four square cylinders of side $\frac{\lambda}{6} = 0.1667$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$ (two squares), $\varepsilon_r = 1.9$ (two square)
- $\sigma = 0$ [S/m]

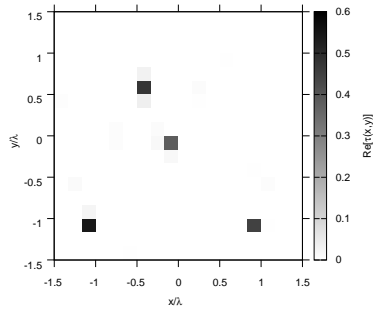
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

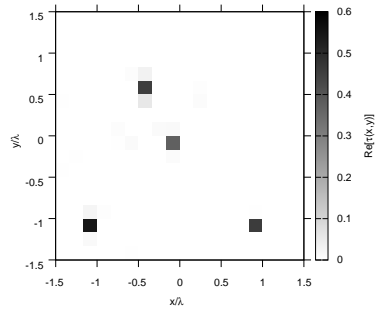
RESULTS: $\varepsilon_r = 1.5$



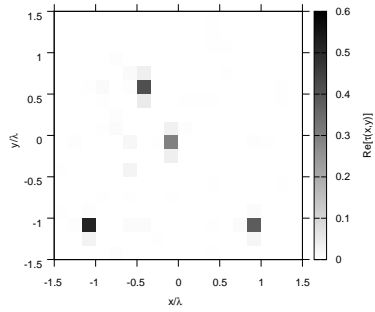
(a)



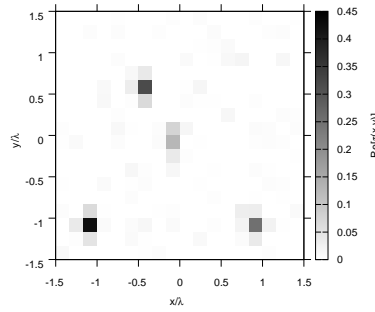
(b)



(c)



(d)



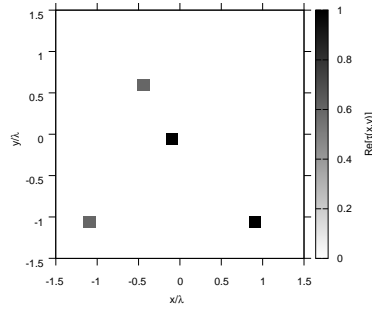
(e)

Figure 27. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB], (d) $SNR = 10$ [dB], (e) $SNR = 5$ [dB].

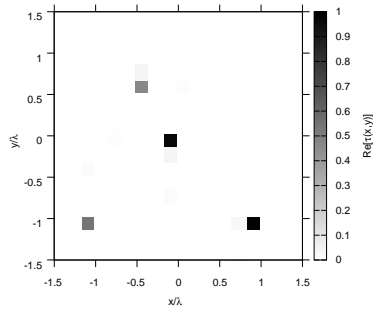
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

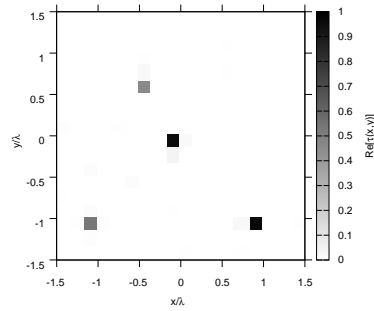
RESULTS: $\varepsilon_r = 2.0$



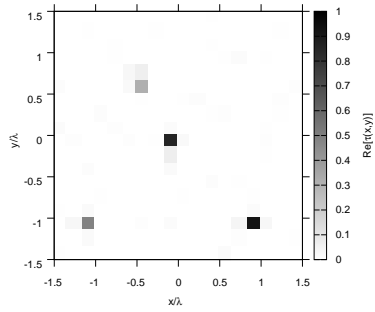
(a)



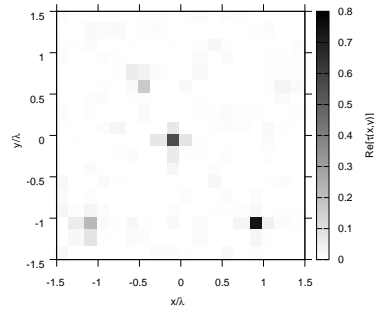
(b)



(c)



(d)



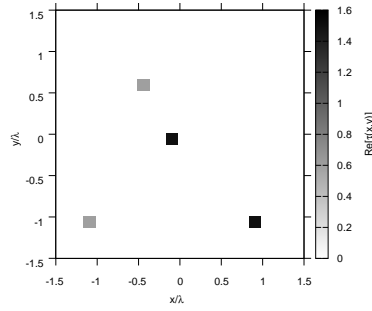
(e)

Figure 28. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

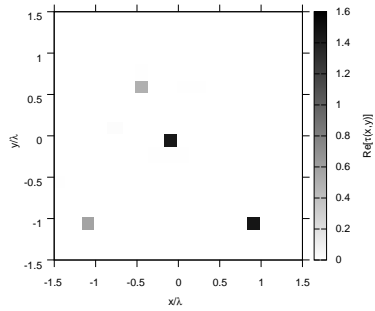
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

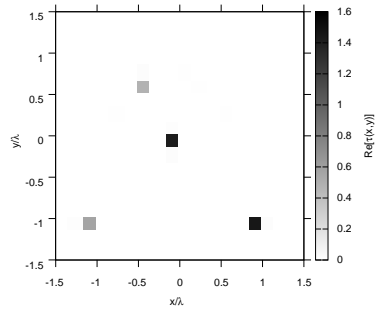
RESULTS: $\varepsilon_r = 2.5$



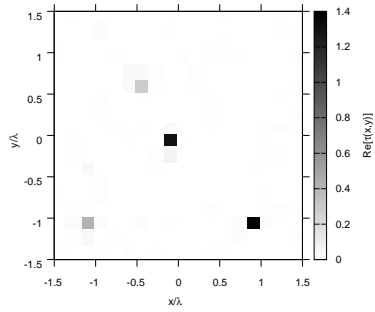
(a)



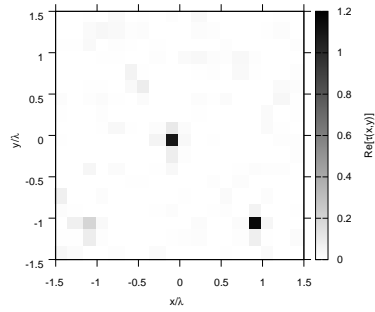
(b)



(c)



(d)



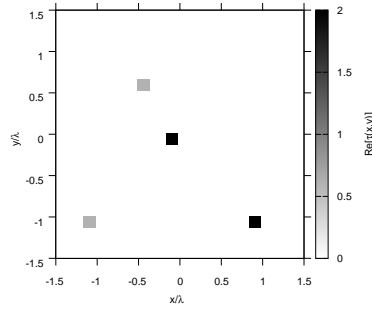
(e)

Figure 29. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB], (d) $SNR = 10$ [dB], (e) $SNR = 5$ [dB].

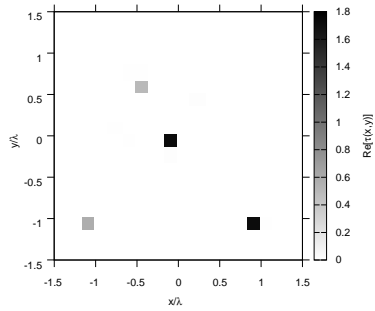
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

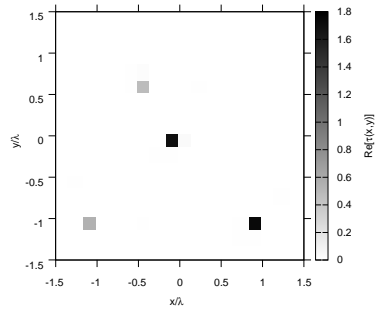
RESULTS: $\varepsilon_r = 3.0$



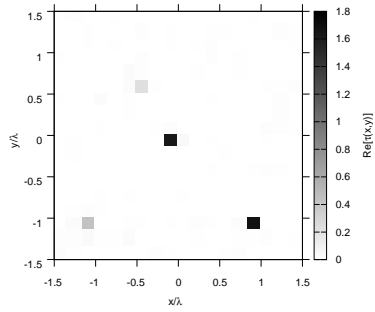
(a)



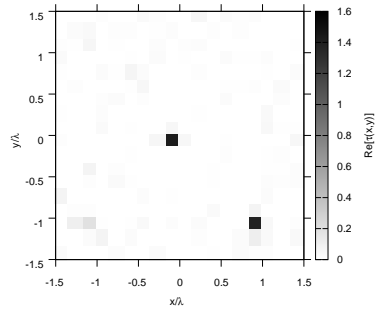
(b)



(c)



(d)



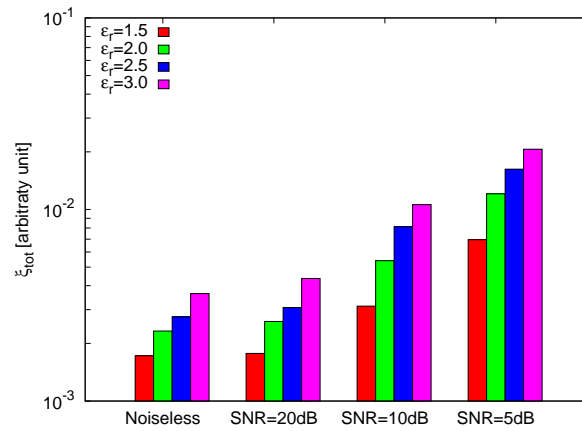
(e)

Figure 30. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

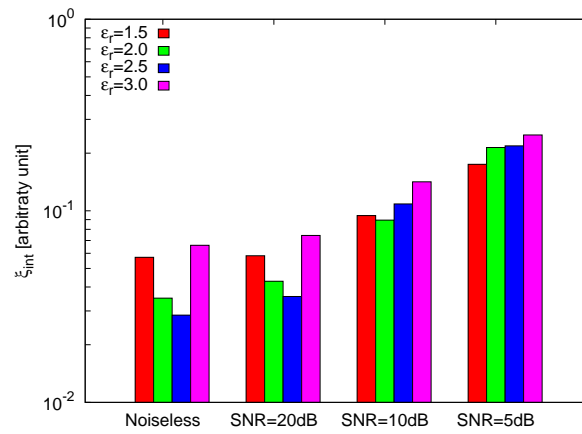
Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20$ dB.

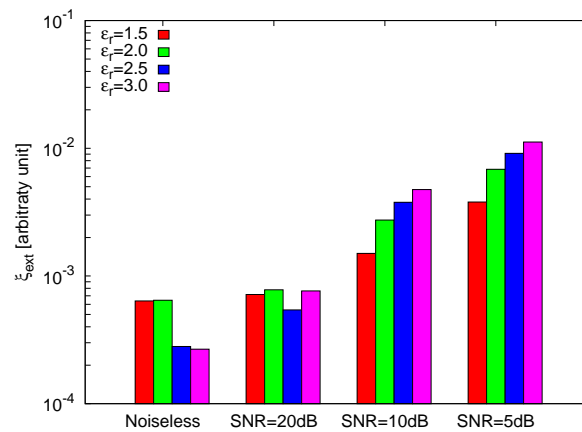
RESULTS: Error Figures



(a)



(b)



(c)

Figure 31. Behaviour of error figures as a function of ϵ_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

Observations:

Ricostruzioni molto buone per i casi Noiseless e $SNR = 20 dB$.

4 TEST CASE: Cross-Shaped Cylinder

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

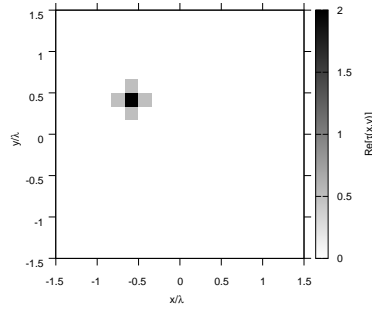
Object:

- Cross-shaped cylinder
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$ [S/m]

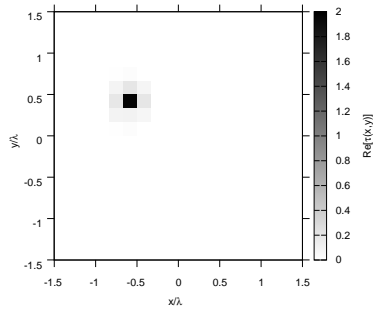
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

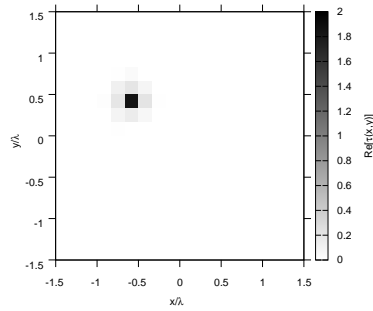
RESULTS: $\varepsilon_r = 1.5$



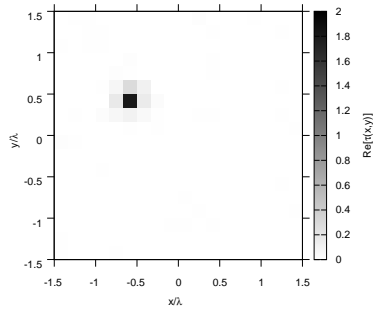
(a)



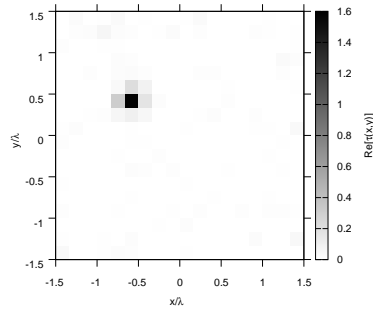
(b)



(c)



(d)



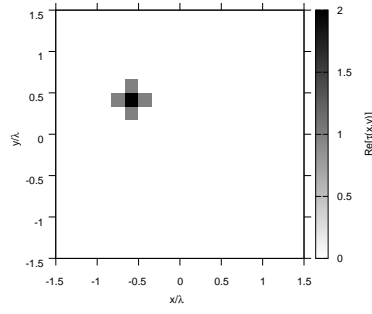
(e)

Figure 32. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

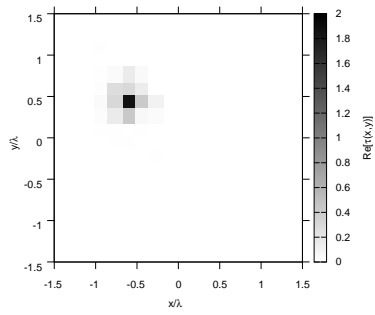
Observations:

Ricostruzioni abbastanza buone solo per $\varepsilon_r = 1.5$.

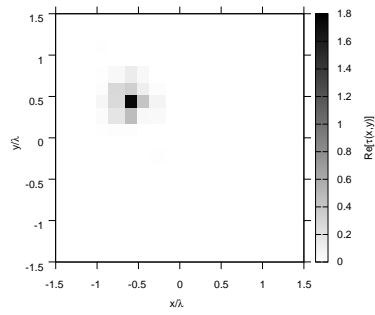
RESULTS: $\varepsilon_r = 2.0$



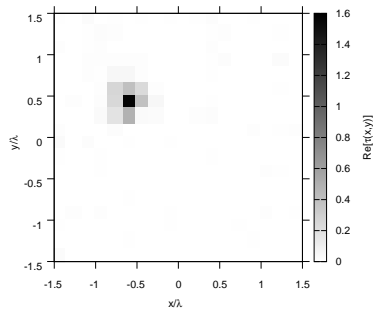
(a)



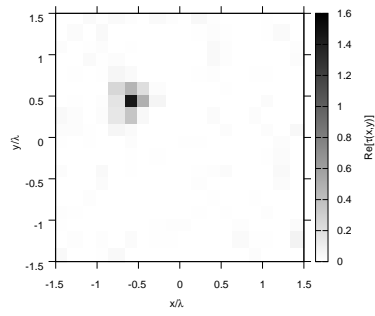
(b)



(c)



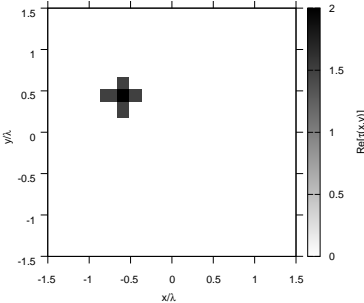
(d)



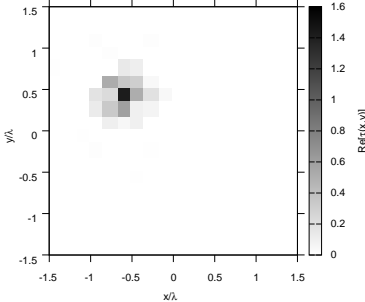
(e)

Figure 33. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

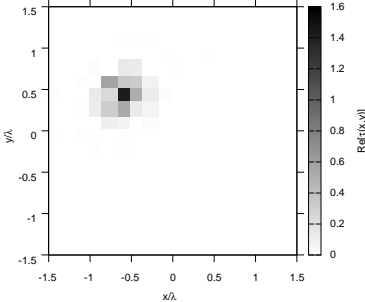
RESULTS: $\epsilon_r = 2.5$



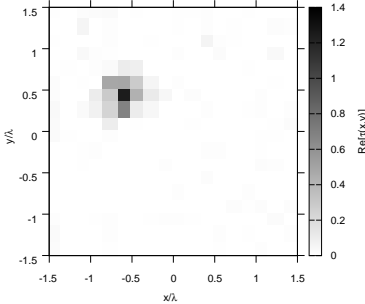
(a)



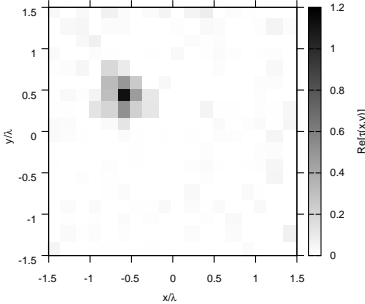
(b)



(c)



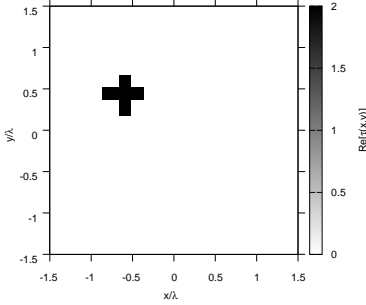
(d)



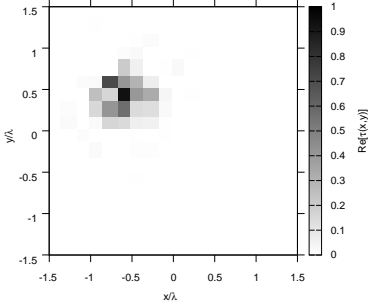
(e)

Figure 34. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

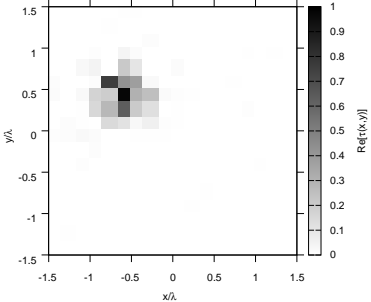
RESULTS: $\epsilon_r = 3.0$



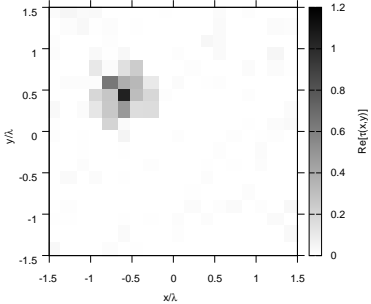
(a)



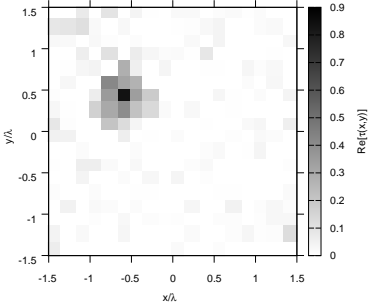
(b)



(c)



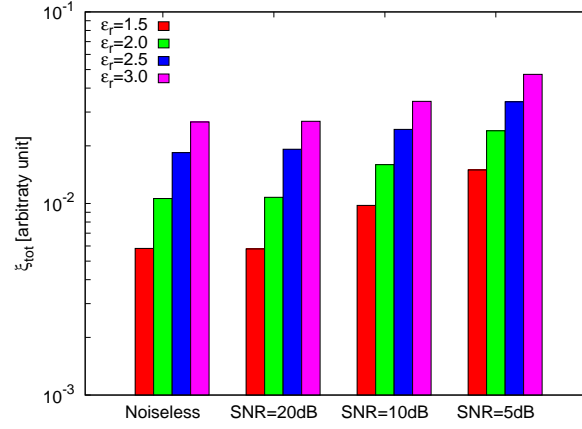
(d)



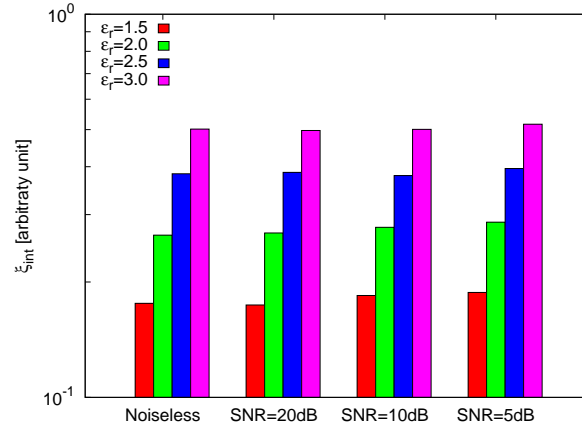
(e)

Figure 35. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

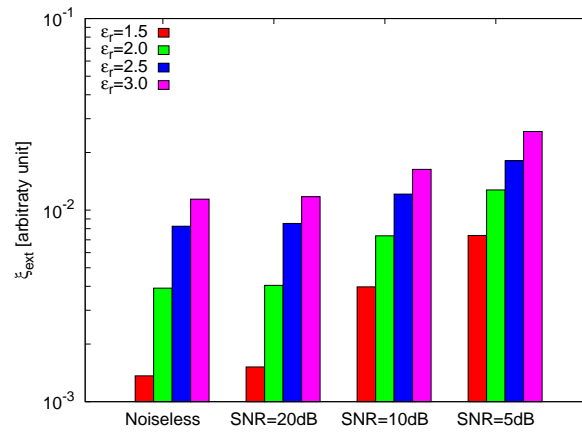
RESULTS: Error Figures



(a)



(b)



(c)

Figure 36. Behaviour of error figures as a function of ϵ_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

5 TEST CASE: L-Shaped Cylinder

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

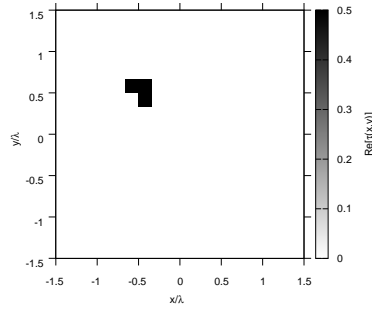
Object:

- L-shaped cylinder
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$ [S/m]

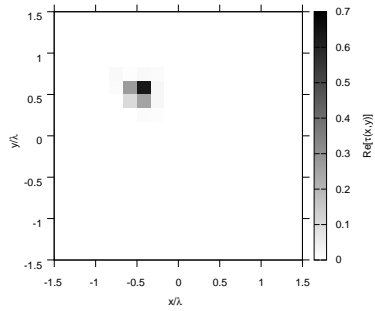
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

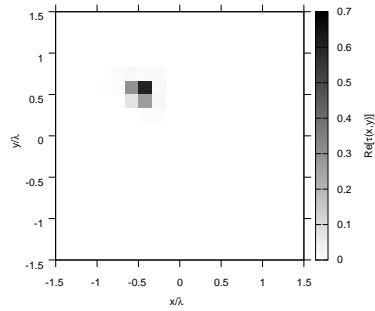
RESULTS: $\varepsilon_r = 1.5$



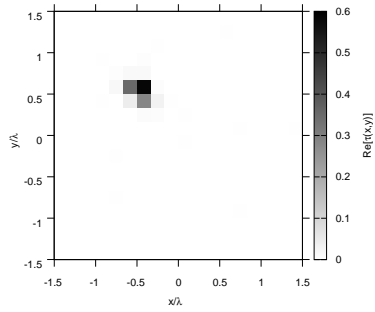
(a)



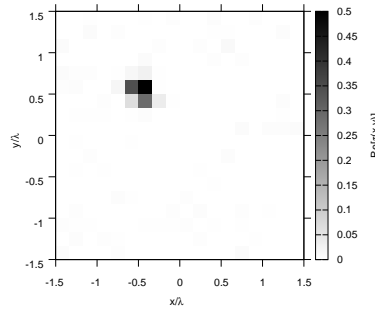
(b)



(c)



(d)



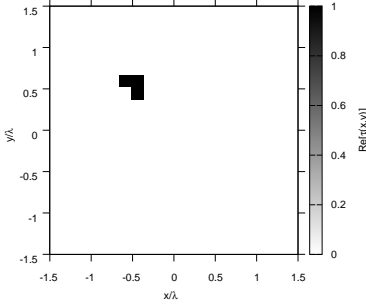
(e)

Figure 37. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB], (d) $SNR = 10$ [dB], (e) $SNR = 5$ [dB].

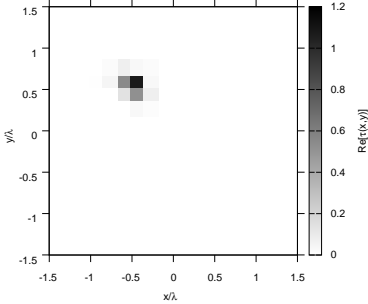
Observations:

Ricostruzioni abbastanza buone solo per $\varepsilon_r = 1.5$.

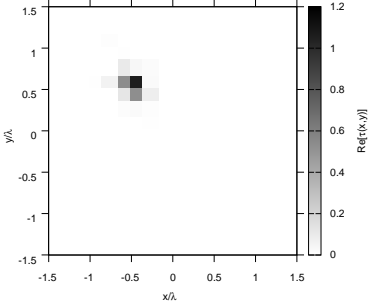
RESULTS: $\epsilon_r = 2.0$



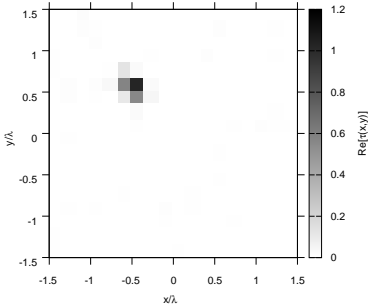
(a)



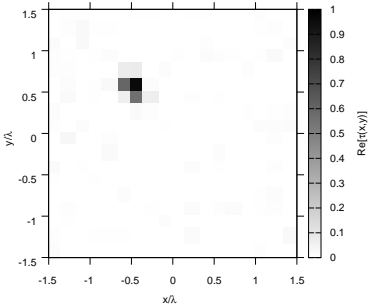
(b)



(c)



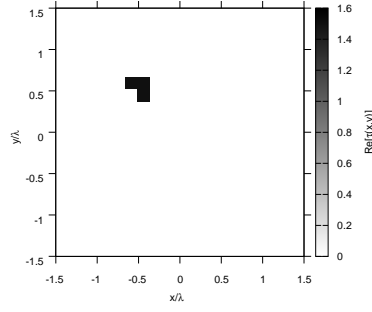
(d)



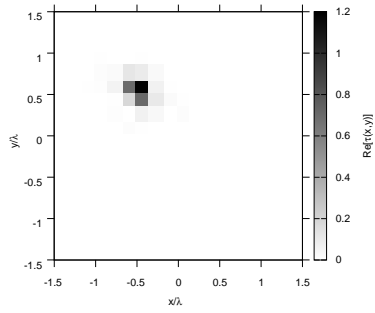
(e)

Figure 38. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

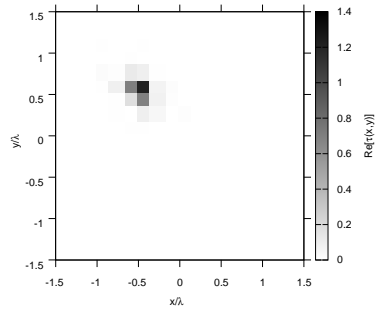
RESULTS: $\varepsilon_r = 2.5$



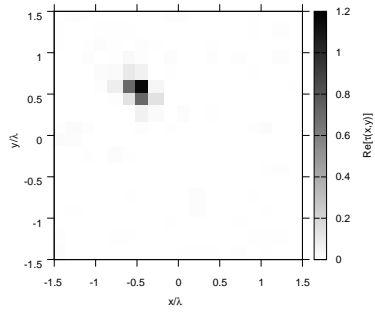
(a)



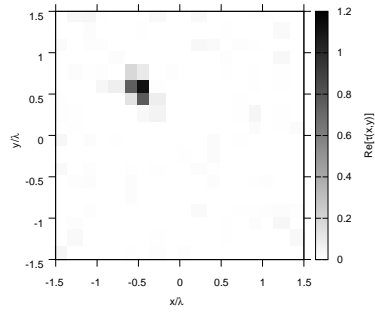
(b)



(c)



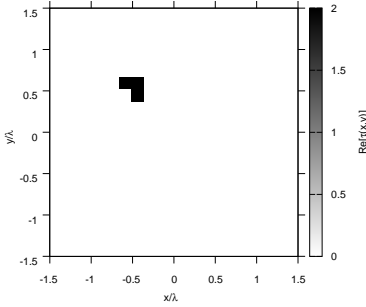
(d)



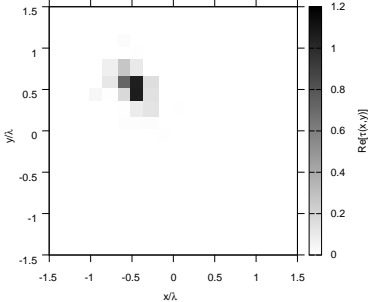
(e)

Figure 39. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

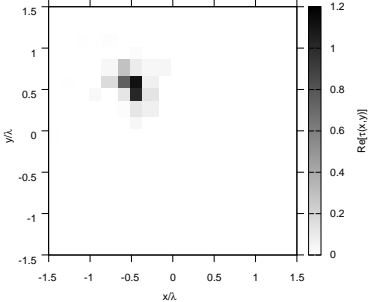
RESULTS: $\epsilon_r = 3.0$



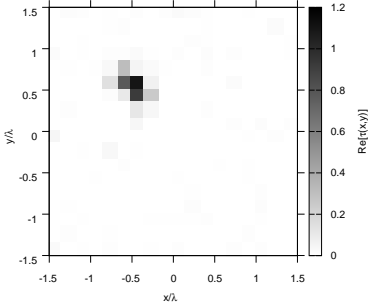
(a)



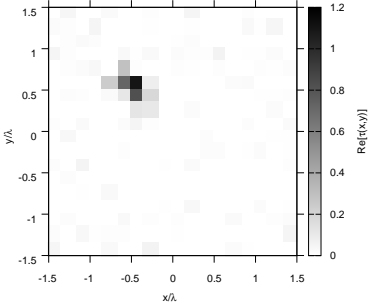
(b)



(c)



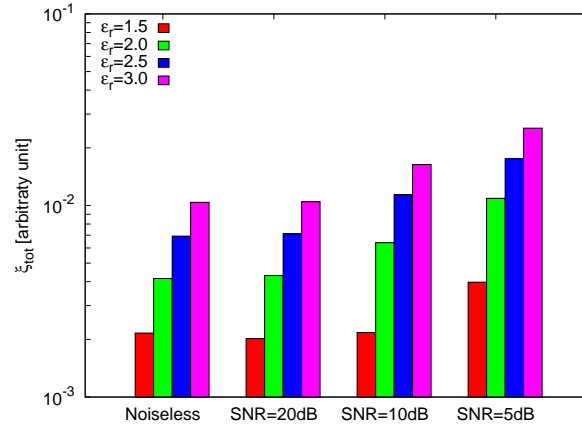
(d)



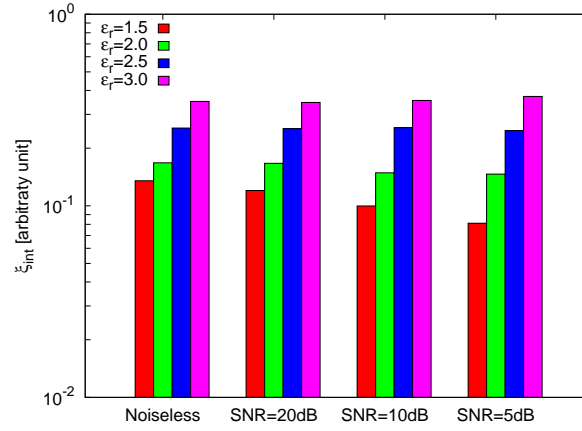
(e)

Figure 40. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

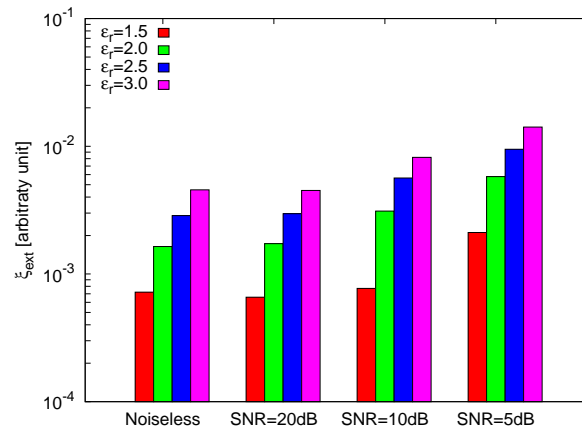
RESULTS: Error Figures



(a)



(b)



(c)

Figure 41. Behaviour of error figures as a function of ε_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

6 TEST CASE: Inhomogeneous L-Shaped Cylinder

GOAL: show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views: V
- Number of Measurements: M
- Number of Cells for the Inversion: N
- Number of Cells for the Direct solver: D
- Side of the investigation domain: L

Test Case Description

Direct solver:

- Square domain divided in $\sqrt{D} \times \sqrt{D}$ cells
- Domain side: $L = 3\lambda$
- $D = 1296$ (discretization for the direct solver: $< \lambda/10$)

Investigation domain:

- Square domain divided in $\sqrt{N} \times \sqrt{N}$ cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- N scelto in modo da essere vicino a $\#DOF$: $N = 324$ (18×18)

Measurement domain:

- Measurement points taken on a circle of radius $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude $A = 1$
- Frequency: 300 MHz ($\lambda = 1$)

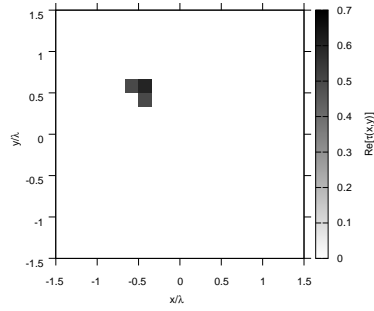
Object:

- Inhomogeneous L-shaped cylinder
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$ [S/m]

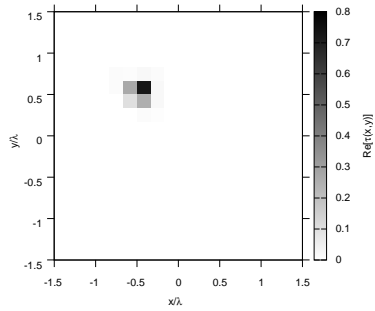
BCS parameters:

- Initial estimate of the noise: $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter: $\tau = 1.0 \times 10^{-8}$

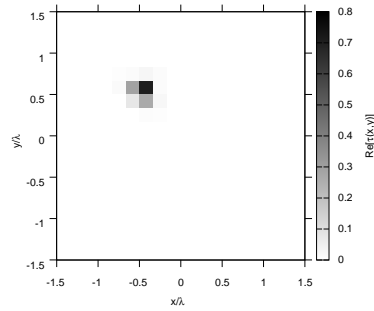
RESULTS: $\varepsilon_r = 1.5$



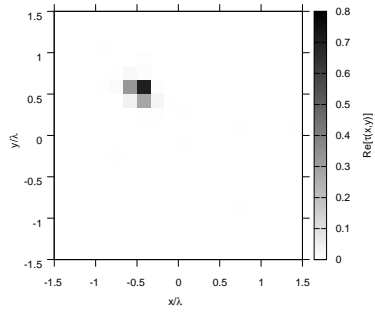
(a)



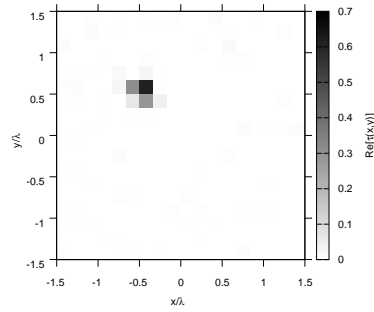
(b)



(c)



(d)



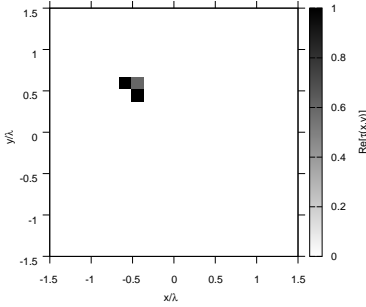
(e)

Figure 42. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

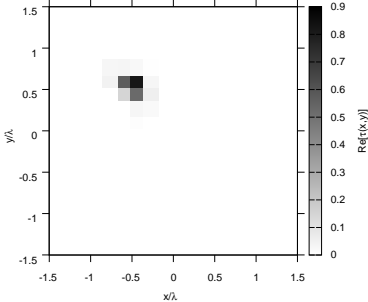
Observations:

Ricostruzioni abbastanza buone solo per $\varepsilon_r = 1.5$.

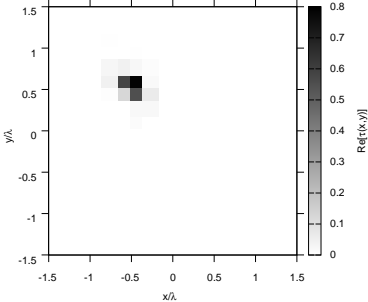
RESULTS: $\varepsilon_r = 2.0$



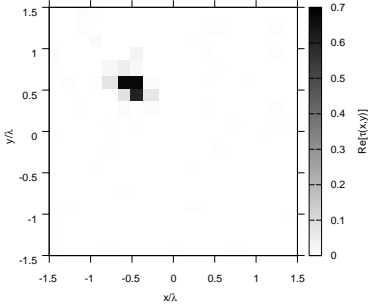
(a)



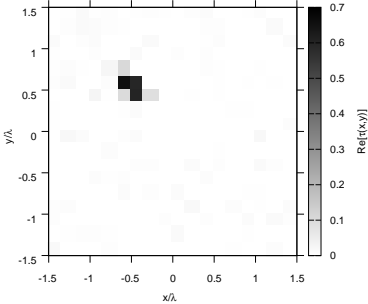
(b)



(c)



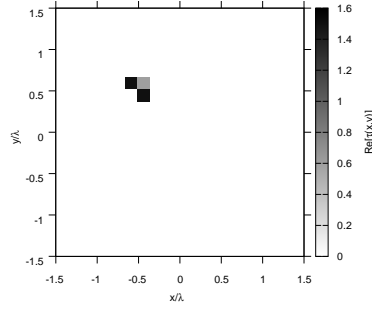
(d)



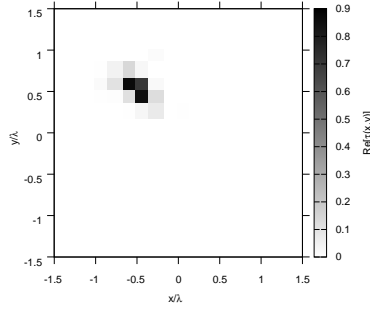
(e)

Figure 43. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

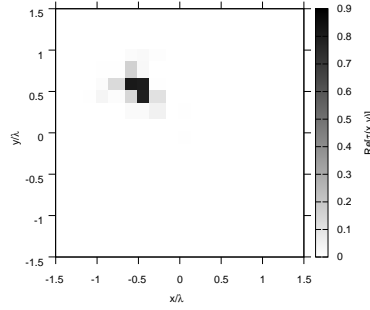
RESULTS: $\varepsilon_r = 2.5$



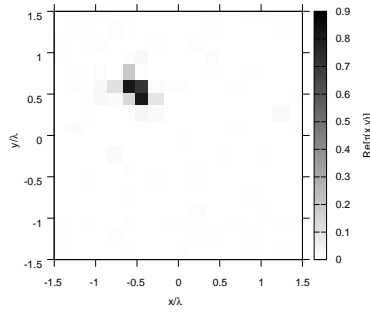
(a)



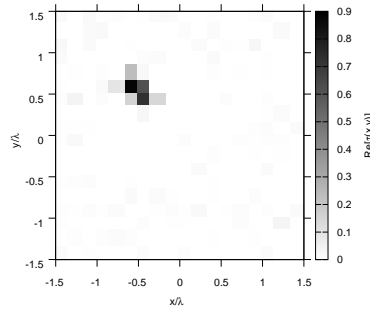
(b)



(c)



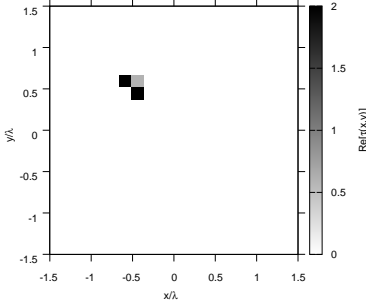
(d)



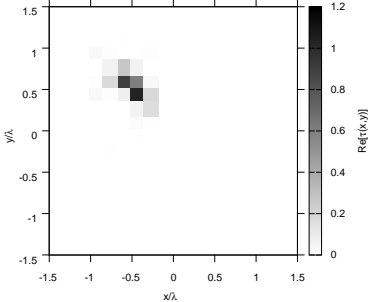
(e)

Figure 44. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

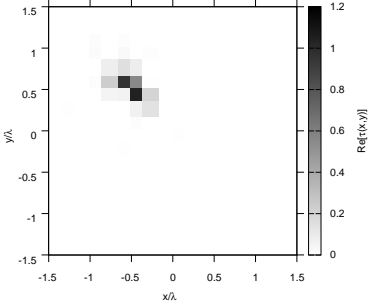
RESULTS: $\epsilon_r = 3.0$



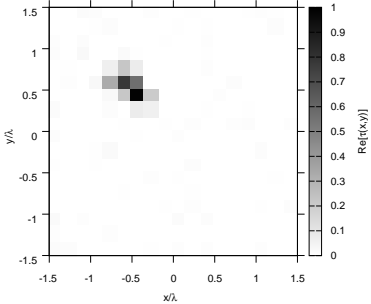
(a)



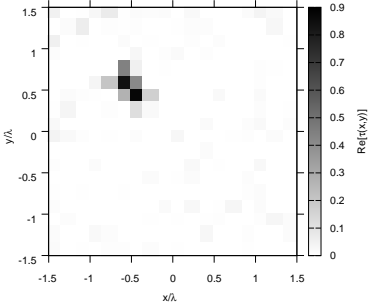
(b)



(c)



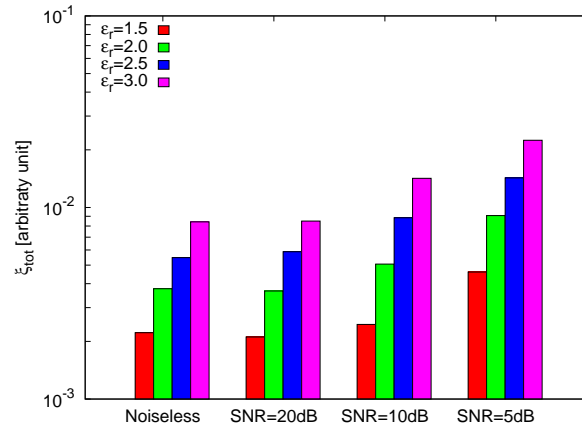
(d)



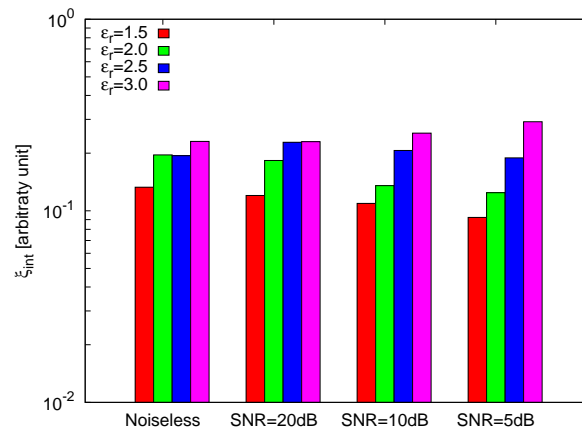
(e)

Figure 45. Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c) $SNR = 20$ [dB] , (d) $SNR = 10$ [dB] , (e) $SNR = 5$ [dB].

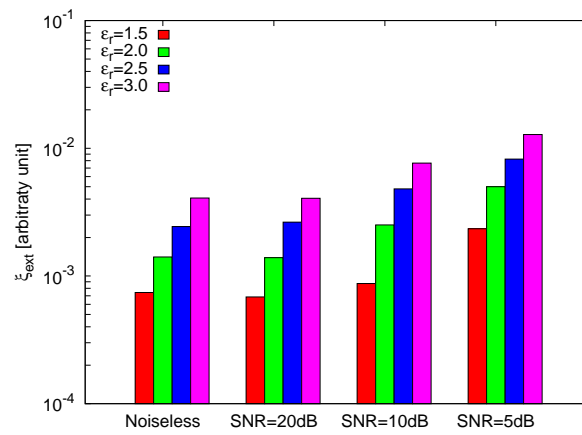
RESULTS: Error Figures



(a)



(b)



(c)

Figure 46. Behaviour of error figures as a function of ε_r , for different SNR values: (a) total error ξ_{tot} , (b) internal error ξ_{int} , (c) external error ξ_{ext} .

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