

# **Preliminary Assessment of a Bayesian Compressive Sampling-based Contrast Field Inversion Method**

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## **Abstract**

In this report, the effectiveness of a novel strategy exploiting the Compressive Sensing paradigm for imaging sparse scatterers at microwave frequencies has been preliminary investigated. After a suitable calibration procedure of the proposed strategy, some introductory results about sparse and weak scatterers have been presented.

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# 1 TEST CASE: Calibration

**GOAL:** show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views:  $V$
- Number of Measurements:  $M$
- Number of Cells for the Inversion:  $N$
- Number of Cells for the Direct solver:  $D$
- Side of the investigation domain:  $L$

## Test Case Description

### Direct solver:

- Square domain divided in  $\sqrt{D} \times \sqrt{D}$  cells
- Domain side:  $L = 3\lambda$
- $D = 1296$  (discretization for the direct solver:  $< \lambda/10$ )

### Investigation domain:

- Square domain divided in  $\sqrt{N} \times \sqrt{N}$  cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- $N$  scelto in modo da essere vicino a  $\#DOF$ :  $N = 324 (18 \times 18)$

### Measurement domain:

- Measurement points taken on a circle of radius  $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

### Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude  $A = 1$
- Frequency: 300 MHz ( $\lambda = 1$ )

### Object:

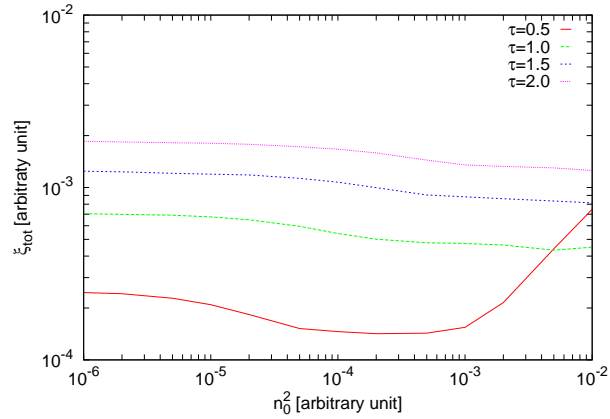
- Square cylinder of side  $\frac{\lambda}{6} = 0.1667$
- Low  $\varepsilon_r$  values:  $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- Very Low  $\varepsilon_r$  values:  $\varepsilon_r \in \{1.1, 1.2, 1.3, 1.4, 1.5\}$

- $\sigma = 0$  [S/m]

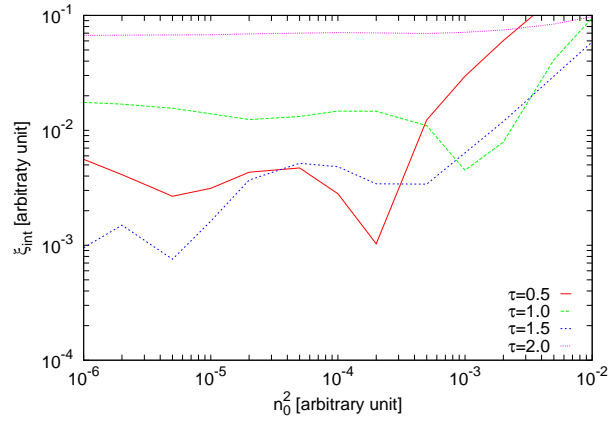
**BCS parameters:**

- Initial estimate of the noise:  $n_0^2 \in \{1.0 \times 10^{-6}, 2.0 \times 10^{-6}, 5.0 \times 10^{-6}, 1.0 \times 10^{-5}, 2.0 \times 10^{-5}, 5.0 \times 10^{-5}, 1.0 \times 10^{-4}, 2.0 \times 10^{-4}, 5.0 \times 10^{-4}, 1.0 \times 10^{-3}, 2.0 \times 10^{-3}, 5.0 \times 10^{-2}, 1.0 \times 10^{-2}, 2.0 \times 10^{-2}, 5.0 \times 10^{-2}, \}$
- Convergence parameter:  $\tau = 10^{-8}$

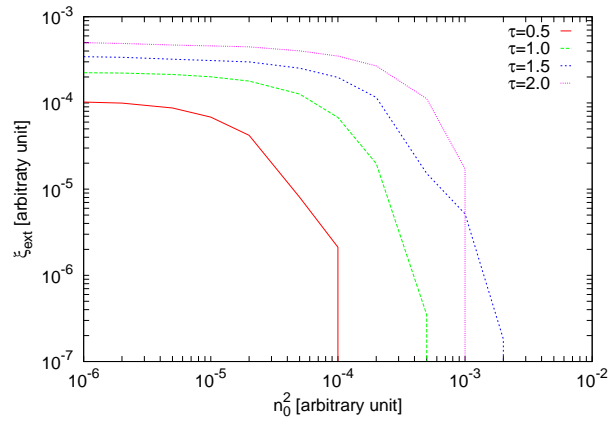
## 1.1 Low $\varepsilon_r$ Values - Noiseless case



(a)



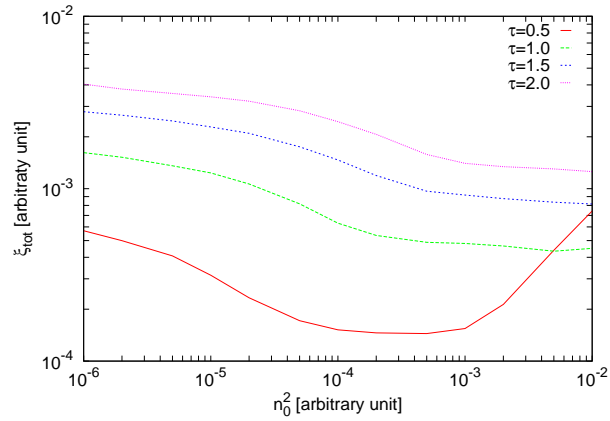
(b)



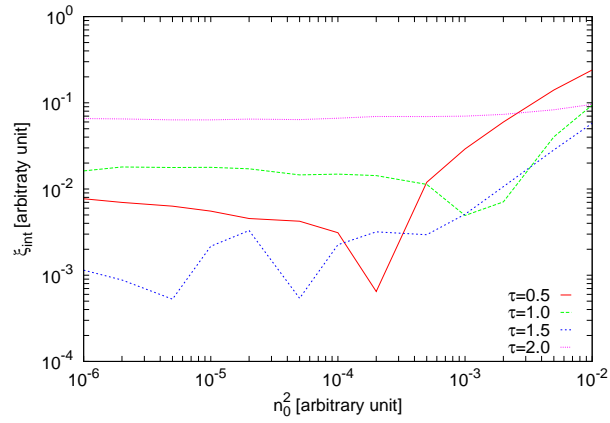
(c)

**Figure 139.** *Calibration* (Noiseless case) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

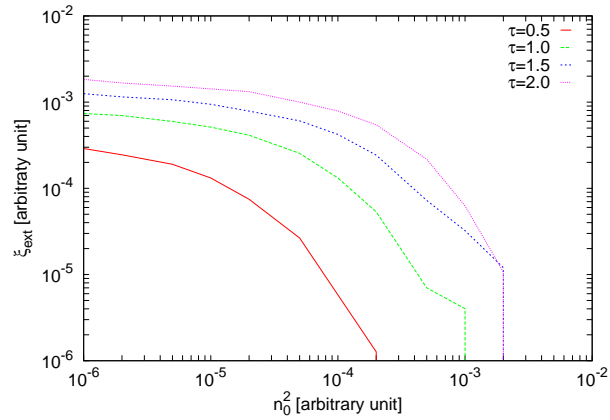
## 1.2 Low $\varepsilon_r$ Values - $SNR = 20$ [dB]



(a)



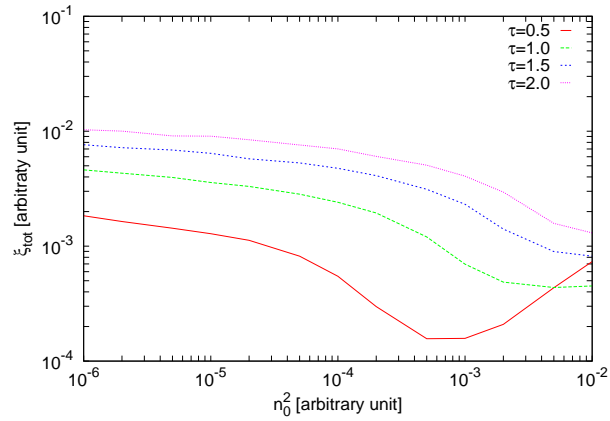
(b)



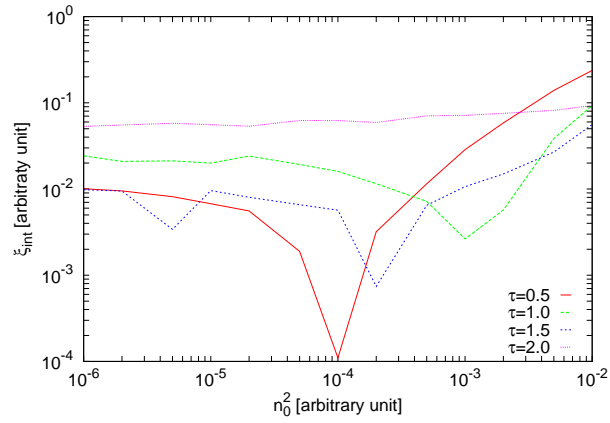
(c)

**Figure 140.** Calibration ( $SNR = 20$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

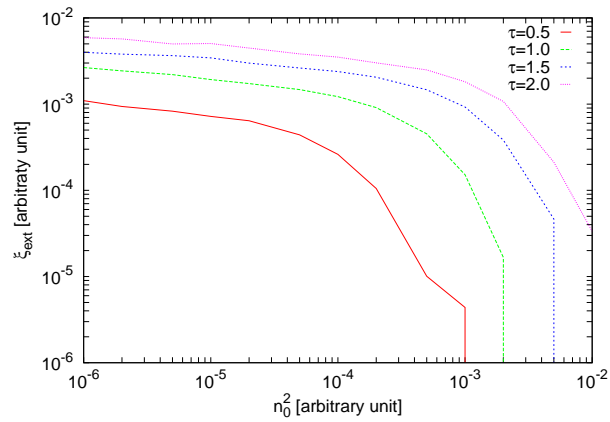
### 1.3 Low $\varepsilon_r$ Values - $SNR = 10$ [dB]



(a)



(b)

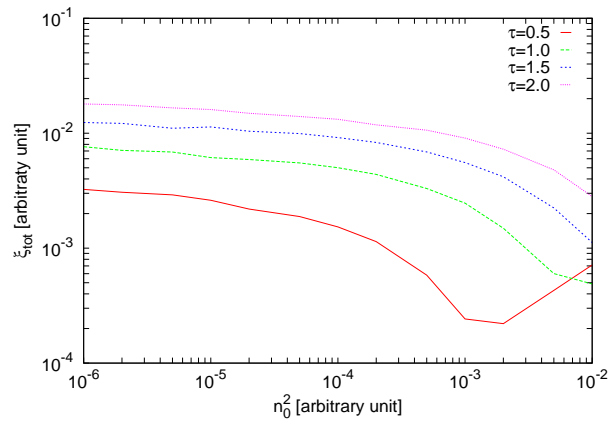


(c)

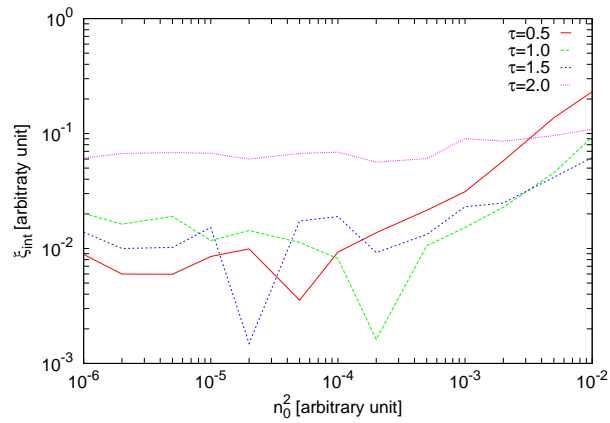
**Figure 141.** Calibration ( $SNR = 10$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .



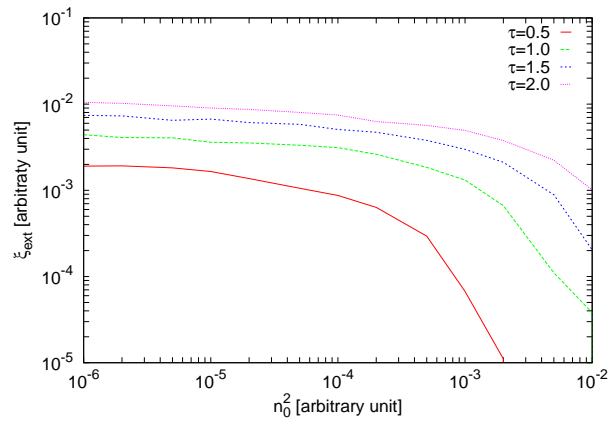
### 1.4 Low $\varepsilon_r$ Values - $SNR = 5$ [dB]



(a)



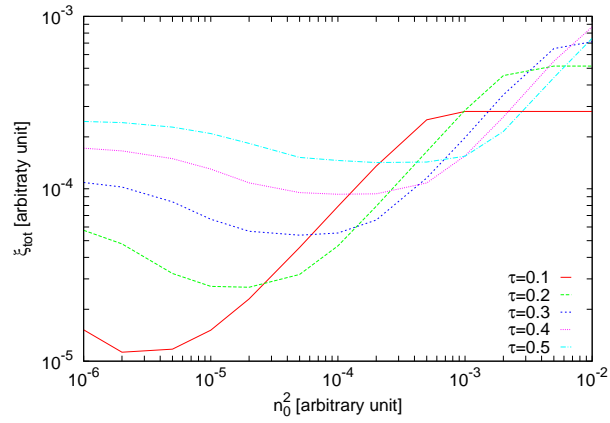
(b)



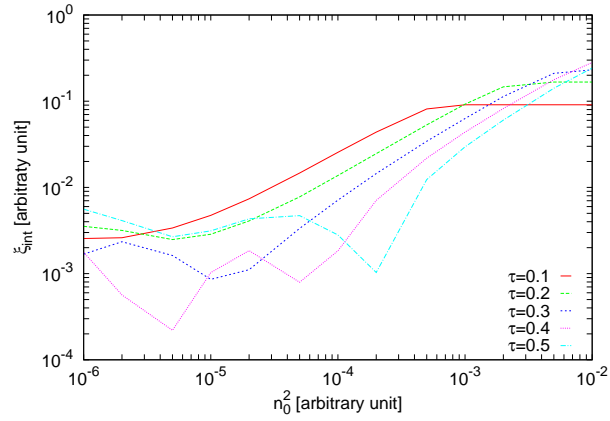
(c)

**Figure 142.** Calibration ( $SNR = 5$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

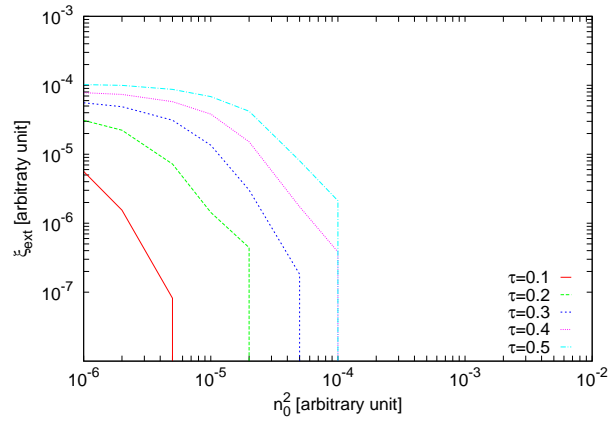
## 1.5 Very Low $\varepsilon_r$ Values - Noiseless case



(a)



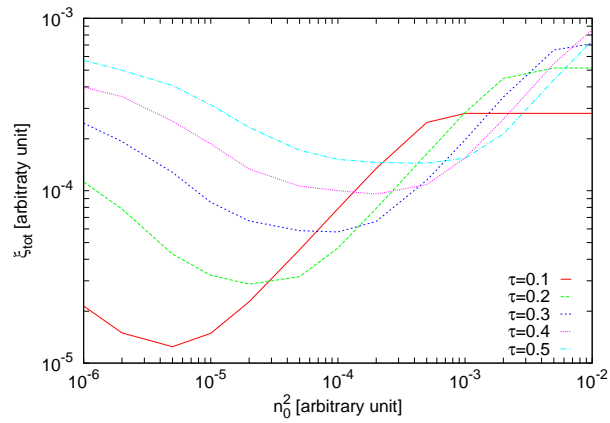
(b)



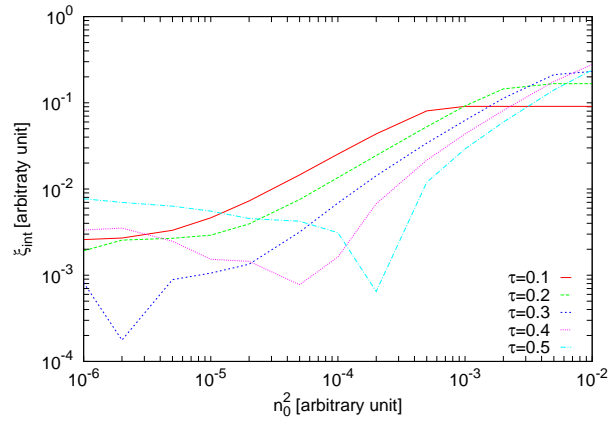
(c)

**Figure 143.** *Calibration* (Noiseless case) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

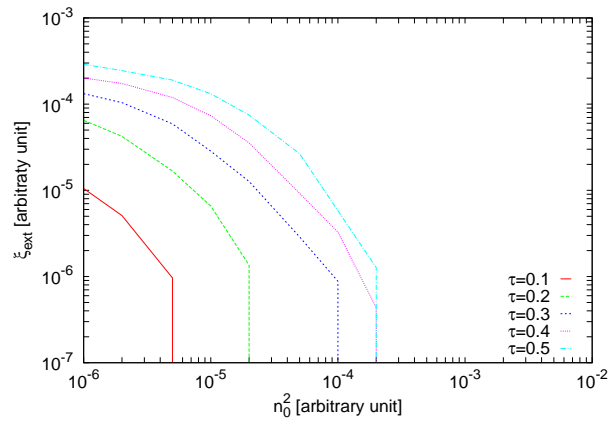
## 1.6 Very Low $\varepsilon_r$ Values - $SNR = 20$ [dB]



(a)



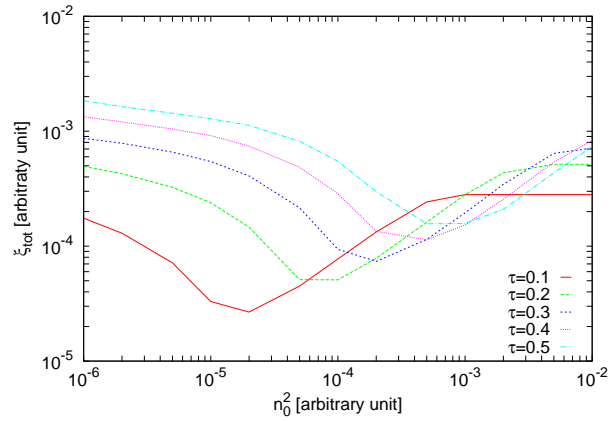
(b)



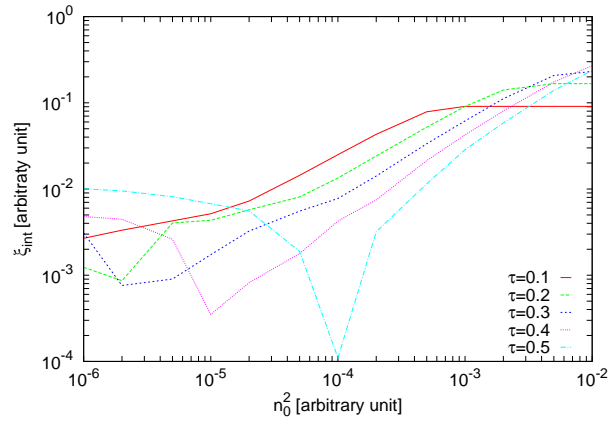
(c)

**Figure 144.** Calibration ( $SNR = 20$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

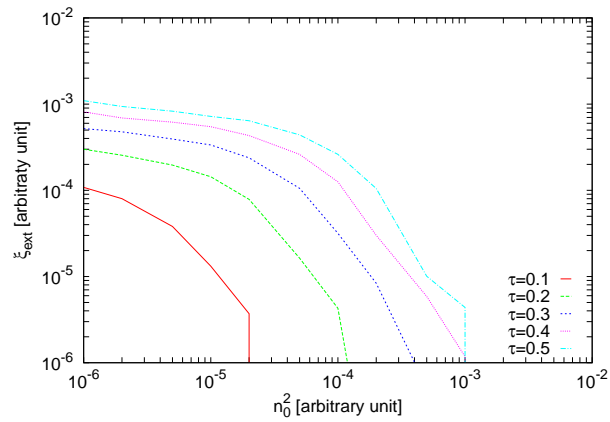
### 1.7 Very Low $\varepsilon_r$ Values - $SNR = 10$ [dB]



(a)



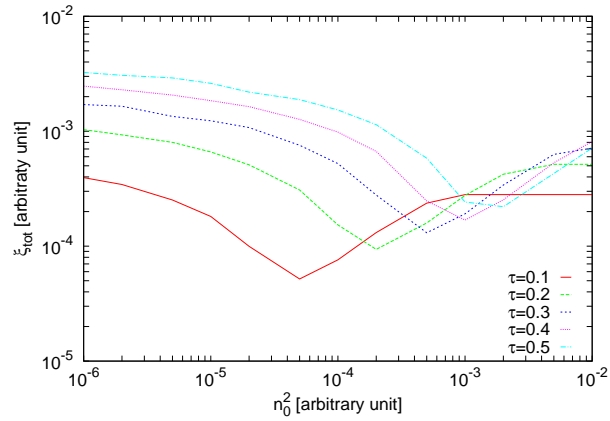
(b)



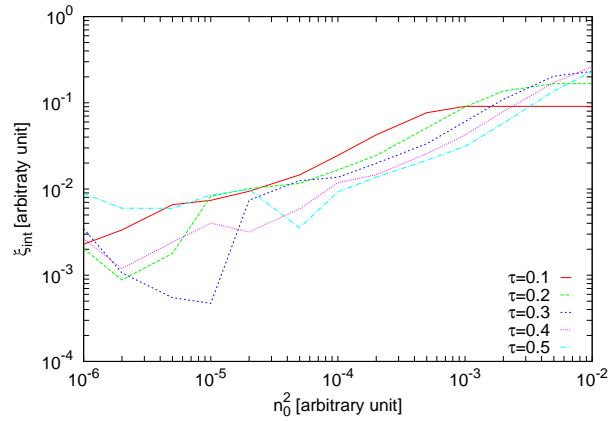
(c)

**Figure 145.** Calibration ( $SNR = 10$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

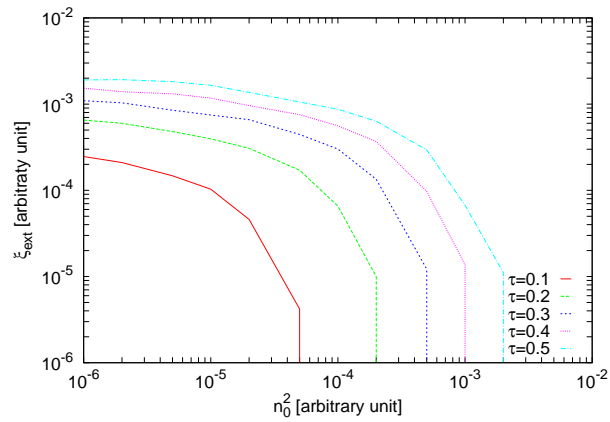
## 1.8 Very Low $\varepsilon_r$ Values - $SNR = 5$ [dB]



(a)



(b)



(c)

**Figure 146.** Calibration ( $SNR = 5$  [dB]) - Behaviour of error figures as a function of  $n_0^2$  and  $\tau$ : (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

## 2 TEST CASE: Square Cylinder $l = 0.16\lambda$

**GOAL:** show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views:  $V$
- Number of Measurements:  $M$
- Number of Cells for the Inversion:  $N$
- Number of Cells for the Direct solver:  $D$
- Side of the investigation domain:  $L$

### Test Case Description

#### Direct solver:

- Square domain divided in  $\sqrt{D} \times \sqrt{D}$  cells
- Domain side:  $L = 3\lambda$
- $D = 1296$  (discretization for the direct solver:  $< \lambda/10$ )

#### Investigation domain:

- Square domain divided in  $\sqrt{N} \times \sqrt{N}$  cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- $N$  scelto in modo da essere vicino a  $\#DOF$ :  $N = 324$  ( $18 \times 18$ )

#### Measurement domain:

- Measurement points taken on a circle of radius  $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

#### Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude  $A = 1$
- Frequency: 300 MHz ( $\lambda = 1$ )

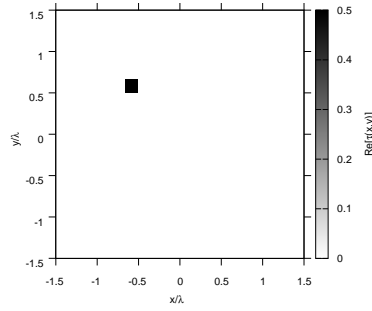
#### Object:

- Square cylinder of side  $\frac{\lambda}{6} = 0.1667$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$  [S/m]

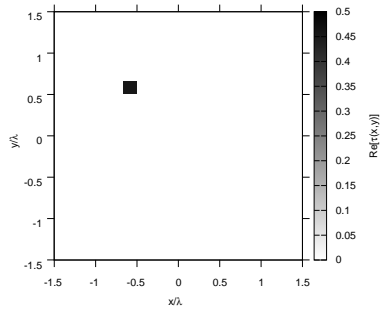
#### BCS parameters:

- Initial estimate of the noise:  $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter:  $\tau = 1.0 \times 10^{-8}$

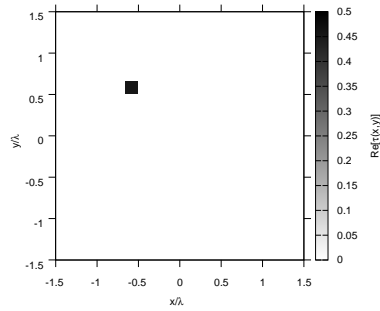
# RESULTS: $\varepsilon_r = 1.5$



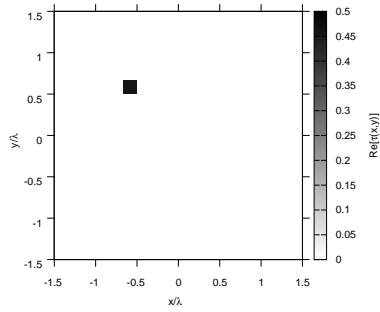
(a)



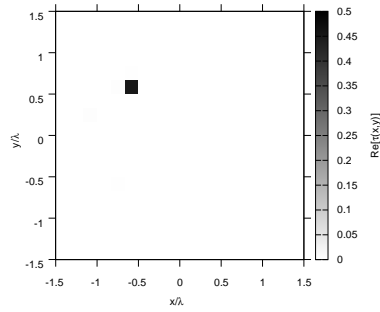
(b)



(c)



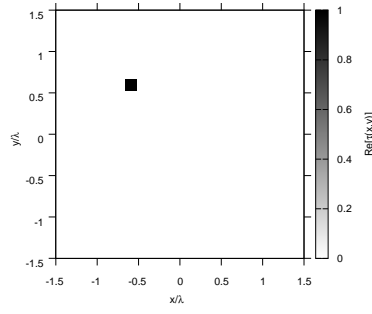
(d)



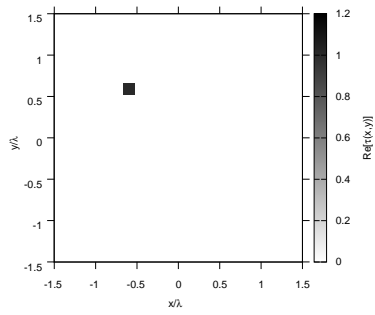
(e)

**Figure 1.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB], (d)  $SNR = 10$  [dB], (e)  $SNR = 5$  [dB].

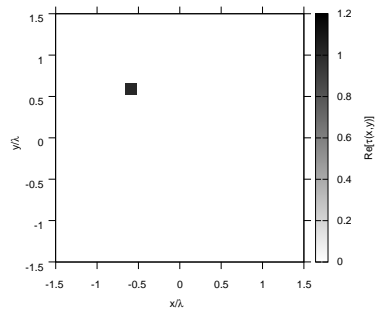
## RESULTS: $\varepsilon_r = 2.0$



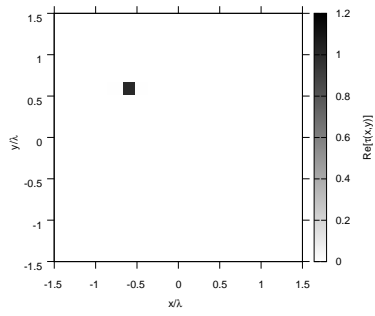
(a)



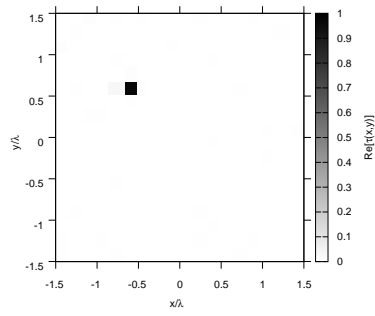
(b)



(c)



(d)

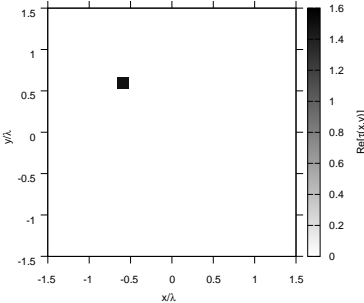


(e)

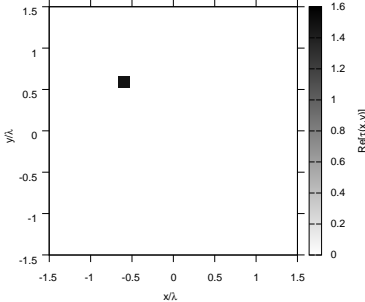
**Figure 2.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].



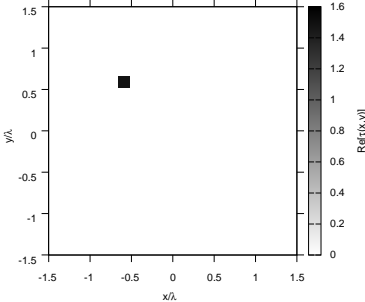
**RESULTS:**  $\epsilon_r = 2.5$



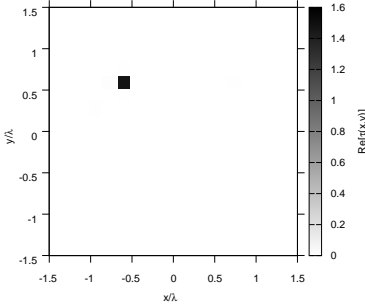
(a)



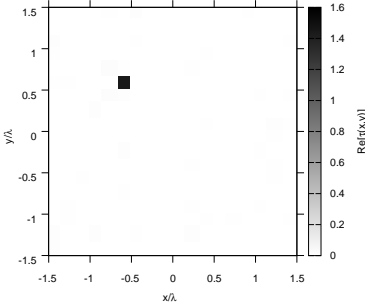
(b)



(c)



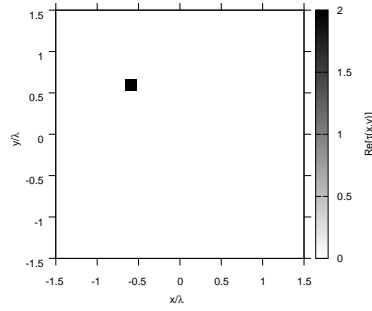
(d)



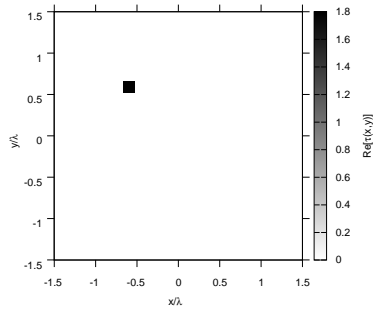
(e)

**Figure 3.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].

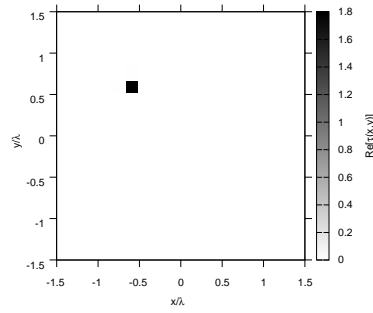
## RESULTS: $\varepsilon_r = 3.0$



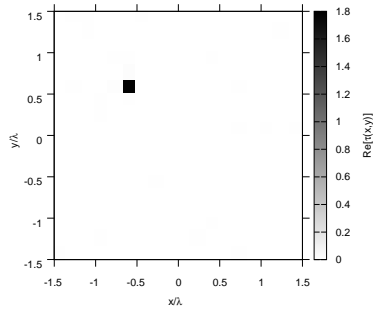
(a)



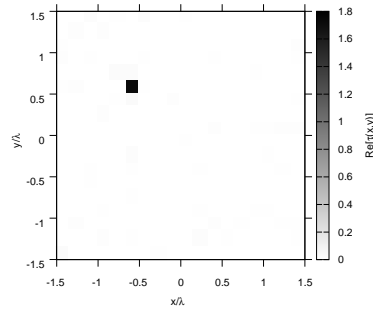
(b)



(c)



(d)



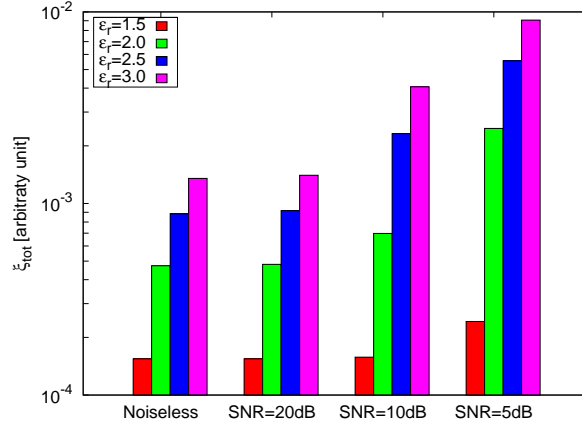
(e)

**Figure 4.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].

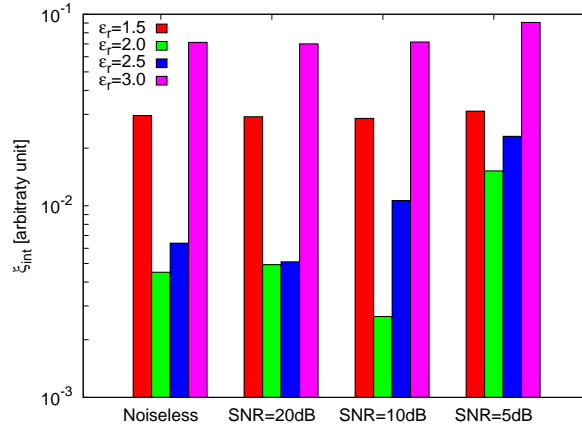
### Observations:

Ricostruzioni molto buone per tutti i valori di  $SNR$ , fino a  $\varepsilon_r = 3.0$ .

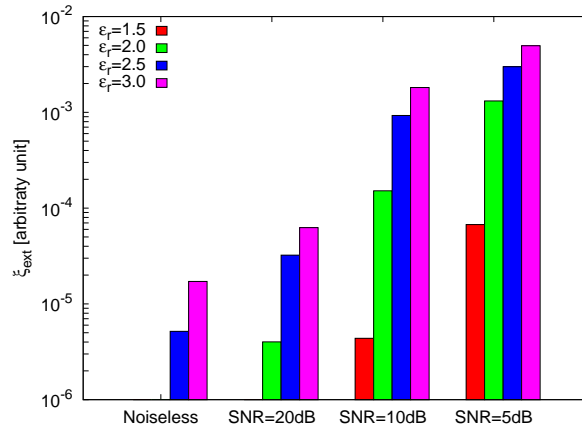
## RESULTS: Error Figures



(a)



(b)



(c)

**Figure 5.** Behaviour of error figures as a function of  $\varepsilon_r$ , for different  $SNR$  values: (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

### Observations:

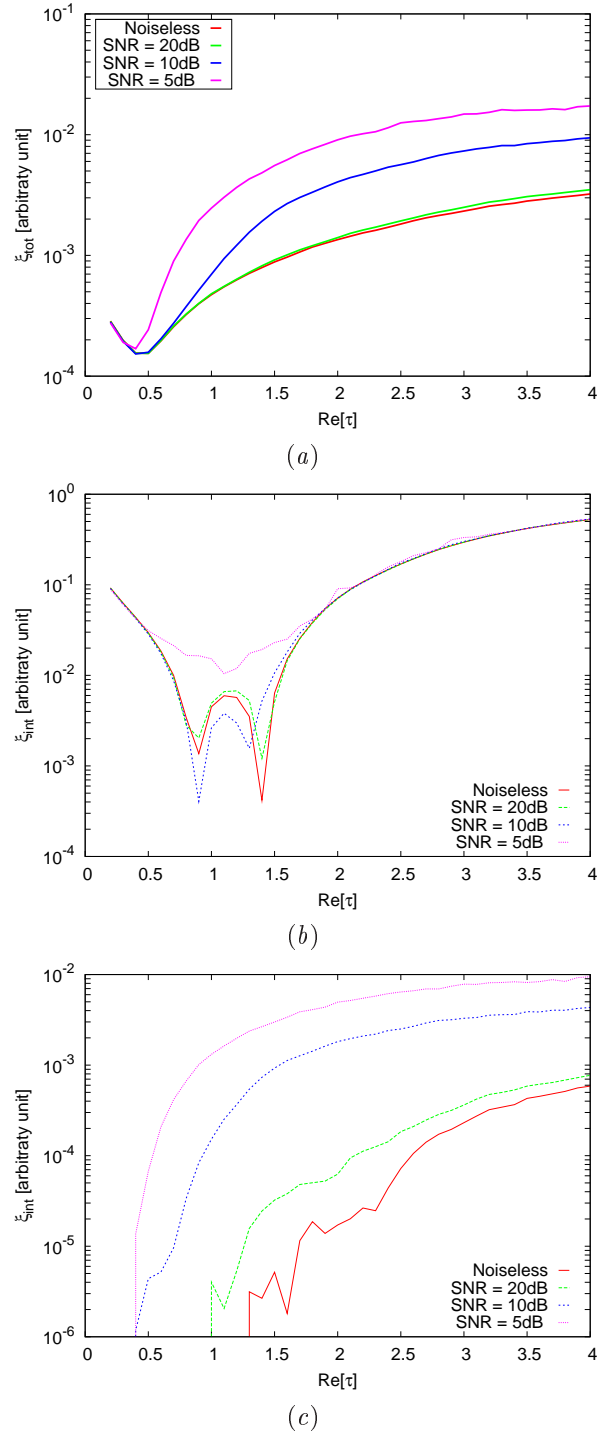
La condizione di validità riportata in [5], ci consente di ricavare il valore di  $\varepsilon_r$  massimo tale per cui è possibile applicare l'approssimazione di Born per il caso in questione, con oggetto scatteratore di dimensione pari a  $\frac{\lambda}{6}$ :

$$n_{\delta}a < \frac{\lambda}{4}$$

dove  $a$  è il raggio dell'oggetto e  $n_{\delta} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$ .

Ne caso in questione otteniamo quindi:  $\sqrt{\varepsilon_r} < \frac{\lambda}{4a} \Rightarrow \sqrt{\varepsilon_r} < \frac{12\lambda}{4\lambda} \Rightarrow \varepsilon_r < 9$ .

## RESULTS: Error Figures

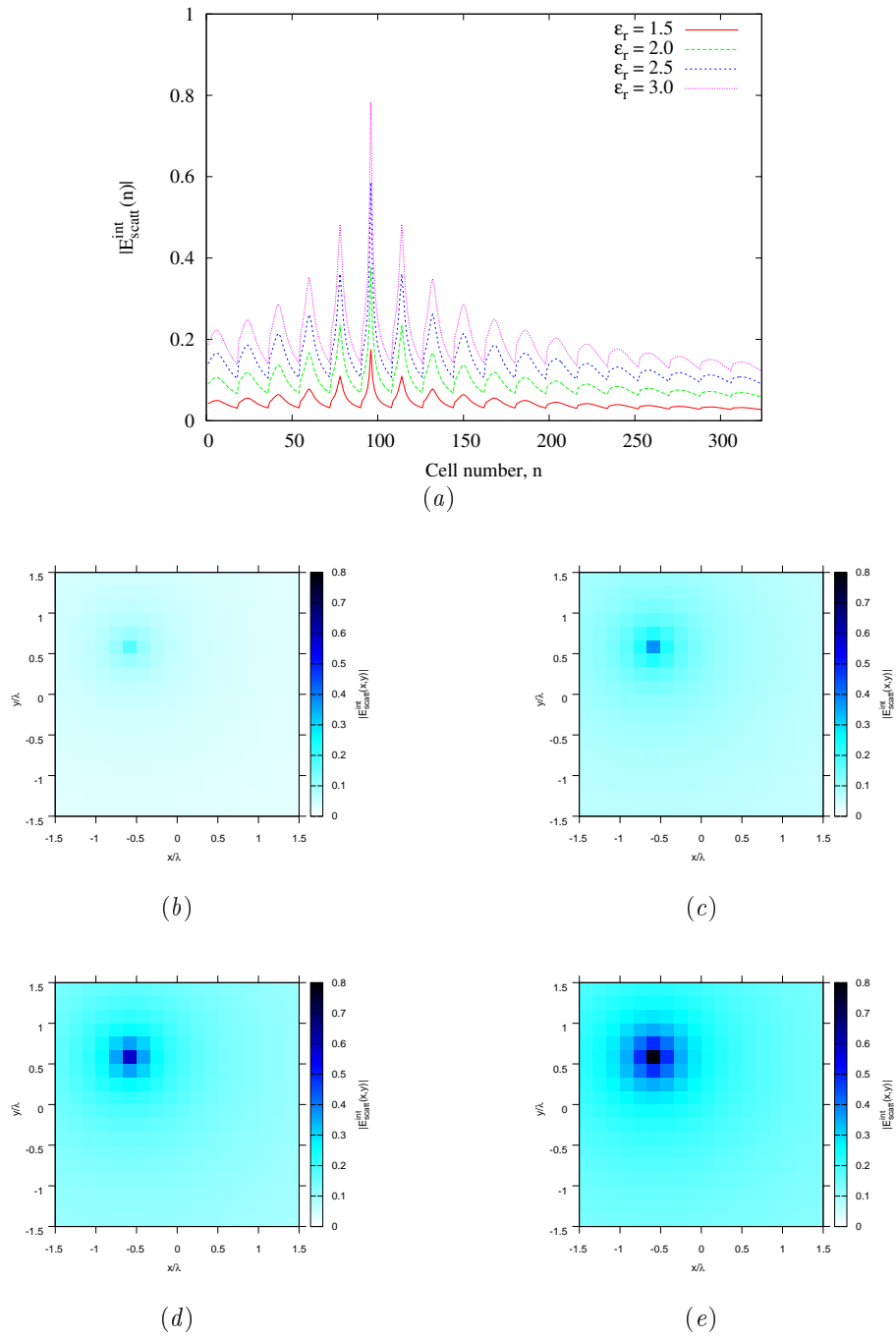


**Figure 6.** Behaviour of error figures as a function of  $\varepsilon_r$ , for different  $SNR$  values: (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

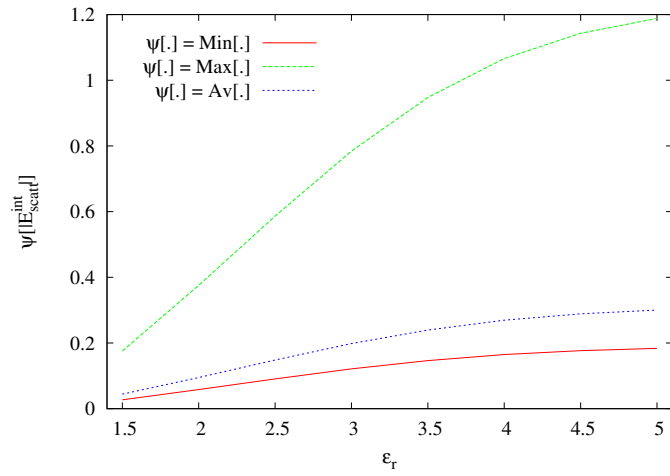
### Observations:

L'errore interno sale vertiginosamente per valori di  $\varepsilon_r$  superiori a 2.5; in generale però, come si può osservare dalle figure delle ricostruzioni, le prestazioni sono buone fino a  $\varepsilon_r = 3.0$ .

# Internal Scattered Field Analysis



**Figure 7.** (a) Average of the absolute value of the scattered field inside the investigation domain for different values of  $\epsilon_r$  : (b)  $\epsilon_r = 1.5$ , (c)  $\epsilon_r = 2.0$ , (d)  $\epsilon_r = 2.5$  and (e)  $\epsilon_r = 3.0$ .



(a)

Figure 8.(a) Internal Scattered Field statistical analysis.

**Observations:**

Le prestazioni della tecnica si possono considerare buone fino a  $\epsilon_r = 3.0$ , ossia, osservando Fig.8.(a), fino a quando  $\frac{E_{tot}^{int} - E_{inc}^{int}}{E_{inc}^{int}} < 0.2$ .

### 3 TEST CASE: Square Cylinder $side = 0.33\lambda$

**GOAL:** show the performances of *BCS* when dealing with a sparse scatterer

- Number of Views:  $V$
- Number of Measurements:  $M$
- Number of Cells for the Inversion:  $N$
- Number of Cells for the Direct solver:  $D$
- Side of the investigation domain:  $L$

#### Test Case Description

##### Direct solver:

- Square domain divided in  $\sqrt{D} \times \sqrt{D}$  cells
- Domain side:  $L = 3\lambda$
- $D = 1296$  (discretization for the direct solver:  $< \lambda/10$ )

##### Investigation domain:

- Square domain divided in  $\sqrt{N} \times \sqrt{N}$  cells
- $L = 3\lambda$
- $2ka = 2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2} = 6\pi\sqrt{2} = 26.65$
- $\#DOF = \frac{(2ka)^2}{2} = \frac{(2 \times \frac{2\pi}{\lambda} \times \frac{L\sqrt{2}}{2})^2}{2} = 4\pi^2 \left(\frac{L}{\lambda}\right)^2 = 4\pi^2 \times 9 \approx 355.3$
- $N$  scelto in modo da essere vicino a  $\#DOF$ :  $N = 324$  ( $18 \times 18$ )

##### Measurement domain:

- Measurement points taken on a circle of radius  $\rho = 3\lambda$
- Full-aspect measurements
- $M \approx 2ka \rightarrow M = 27$

##### Sources:

- Plane waves
- $V \approx 2ka \rightarrow V = 27$
- Amplitude  $A = 1$
- Frequency: 300 MHz ( $\lambda = 1$ )

##### Object:

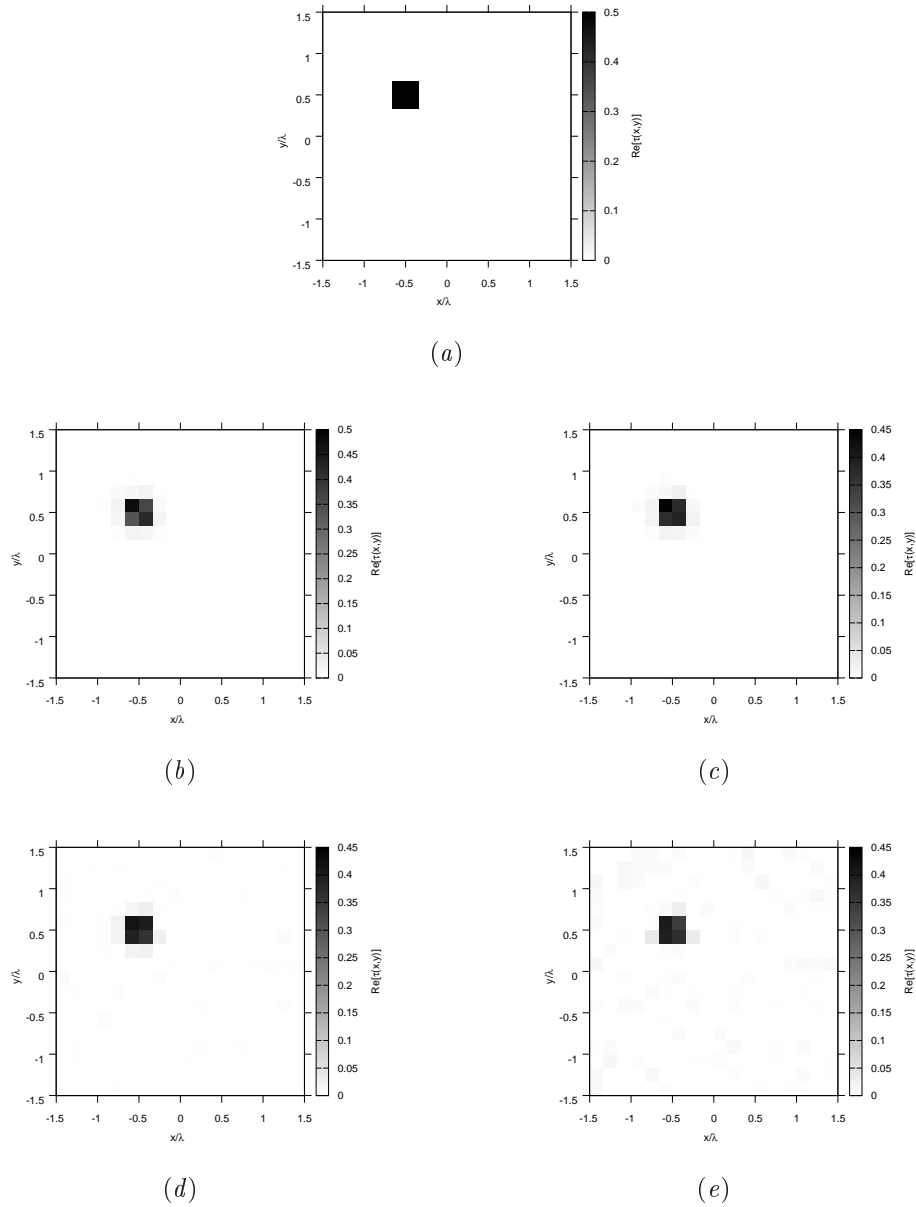
- Square cylinder of side  $\frac{\lambda}{3} = 0.33$
- $\varepsilon_r \in \{1.5, 2.0, 2.5, 3.0\}$
- $\sigma = 0$  [S/m]

##### BCS parameters:

- Initial estimate of the noise:  $n_0 = 1.0 \times 10^{-3}$
- Convergence parameter:  $\tau = 1.0 \times 10^{-8}$



## RESULTS: $\varepsilon_r = 1.5$

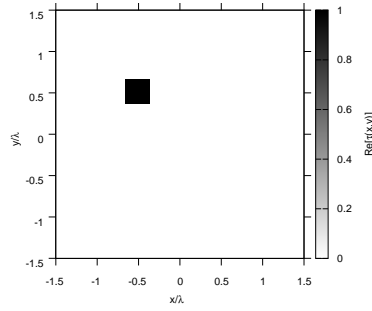


**Figure 9.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB], (d)  $SNR = 10$  [dB], (e)  $SNR = 5$  [dB].

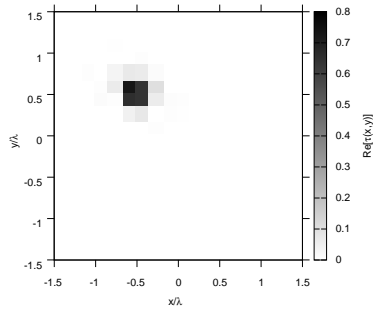
### Observations:

Ricostruzioni buone per i casi Noiseless,  $SNR = 20$  dB e  $SNR = 10$  dB; compare del rumore di fondo per il caso  $SNR = 5$  dB.

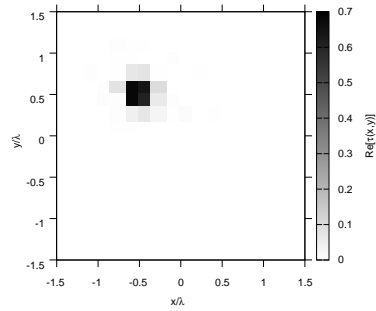
## RESULTS: $\varepsilon_r = 2.0$



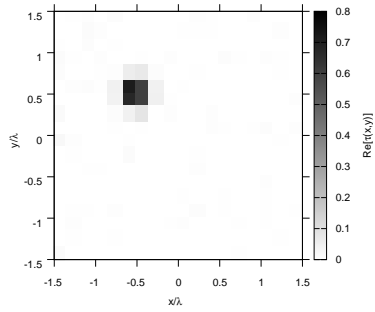
(a)



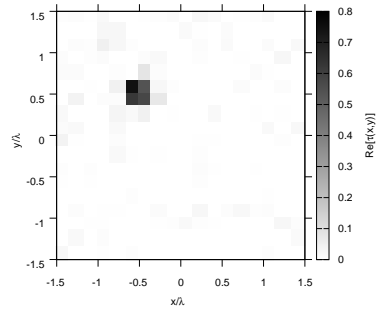
(b)



(c)



(d)



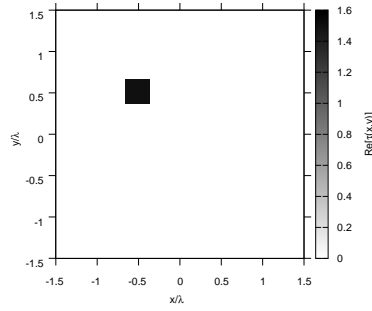
(e)

**Figure 10.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].

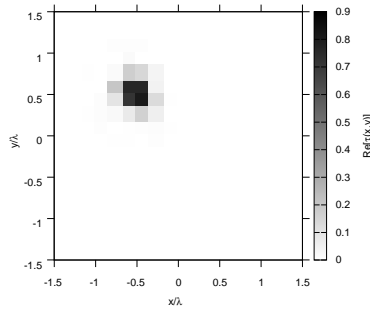
### Observations:

Ricostruzioni abbastanza buone per i casi Noiseless,  $SNR = 20$  dB e  $SNR = 10$  dB; compare del rumore di fondo per il caso  $SNR = 5$  dB.

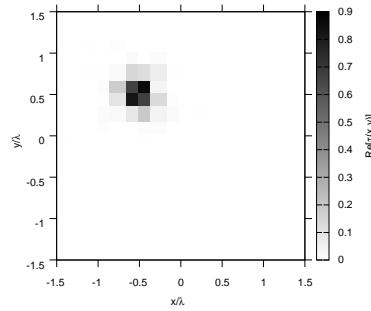
## RESULTS: $\varepsilon_r = 2.5$



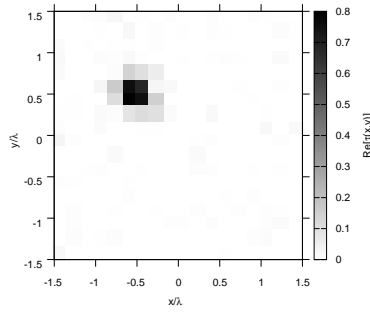
(a)



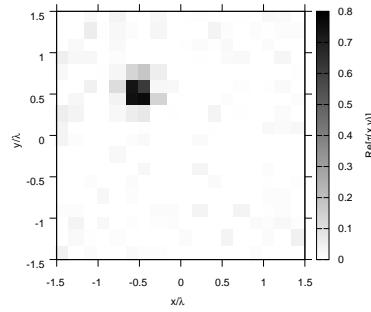
(b)



(c)



(d)



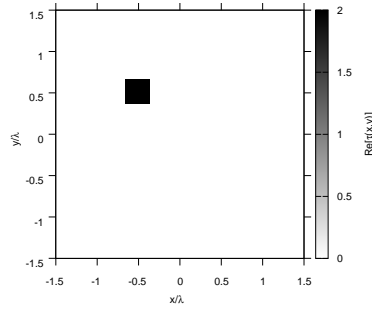
(e)

**Figure 11.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].

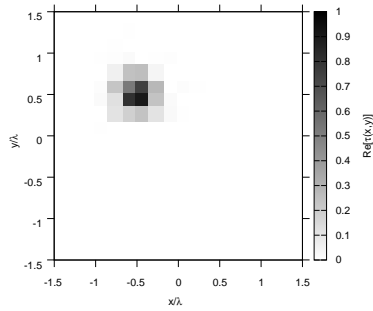
### Observations:

Ricostruzioni sempre più degradate: approssimazione di Born non più applicabile.

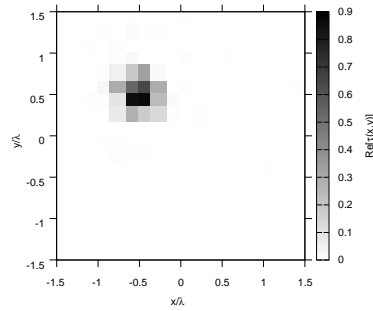
## RESULTS: $\varepsilon_r = 3.0$



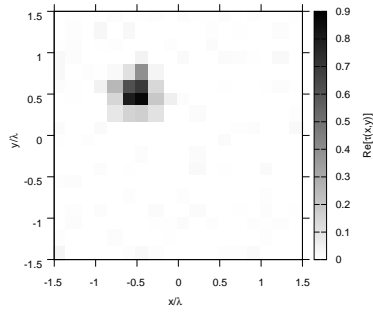
(a)



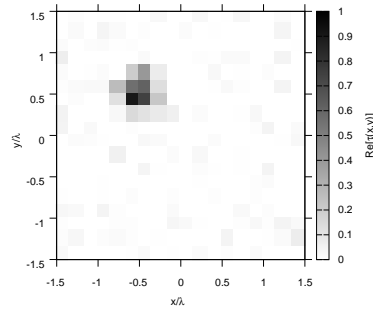
(b)



(c)



(d)



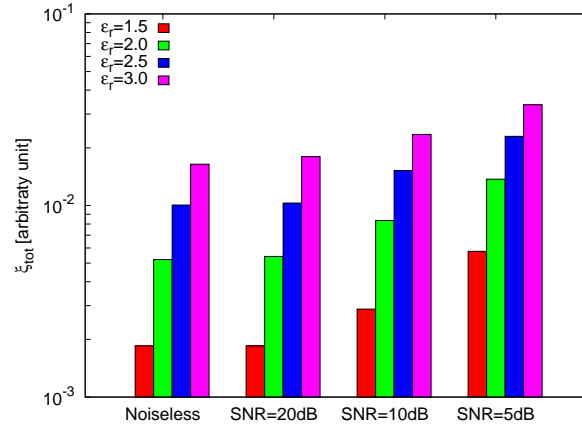
(e)

**Figure 12.** Actual object (a) and BCS reconstructed object for (b) Noiseless case, (c)  $SNR = 20$  [dB] , (d)  $SNR = 10$  [dB] , (e)  $SNR = 5$  [dB].

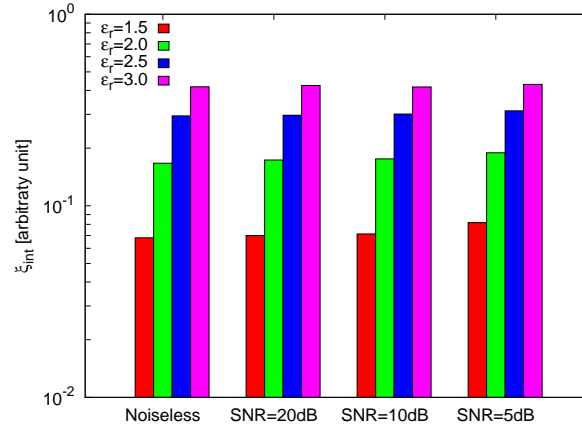
### Observations:

Ricostruzioni sempre più degradate: approssimazione di Born non più applicabile.

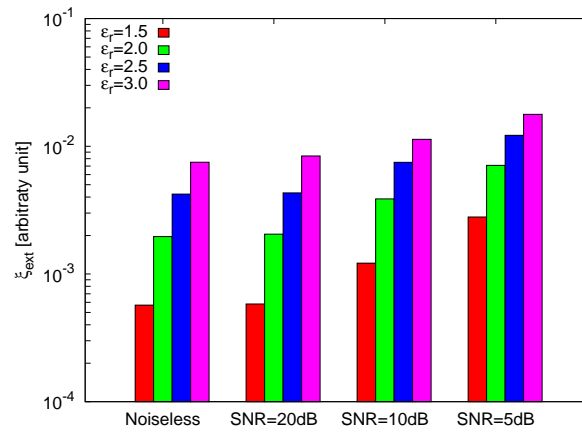
## RESULTS: Error Figures



(a)



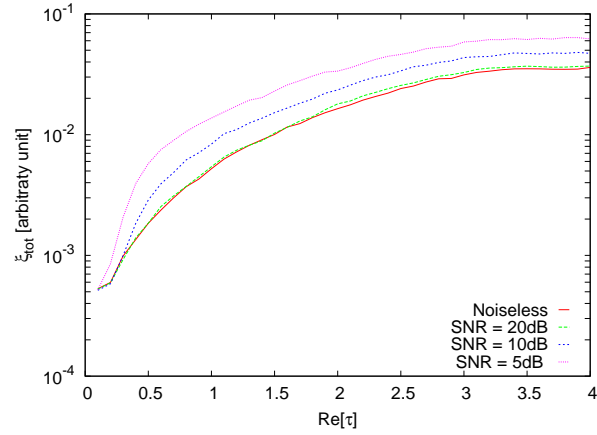
(b)



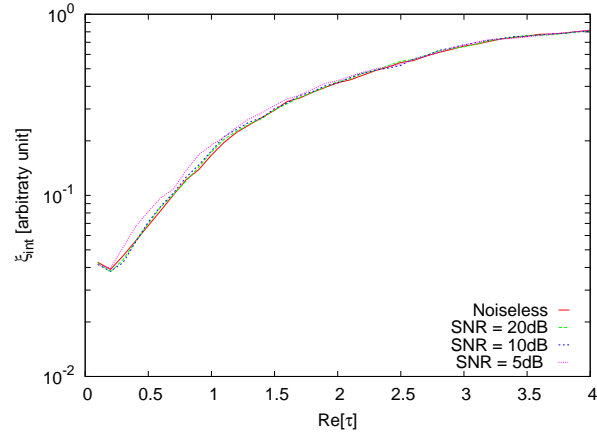
(c)

**Figure 13.** Behaviour of error figures as a function of  $\varepsilon_r$ , for different  $SNR$  values: (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

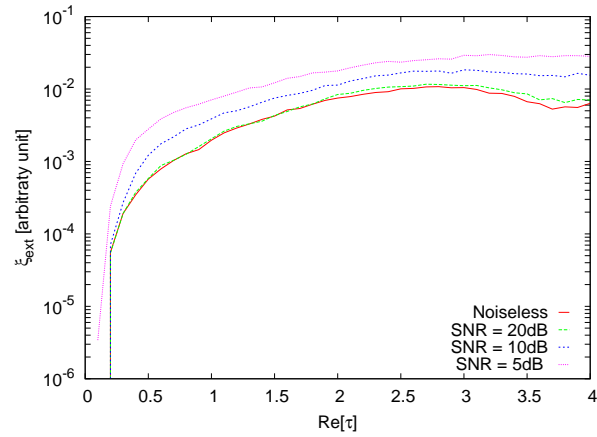
## RESULTS: Error Figures



(a)



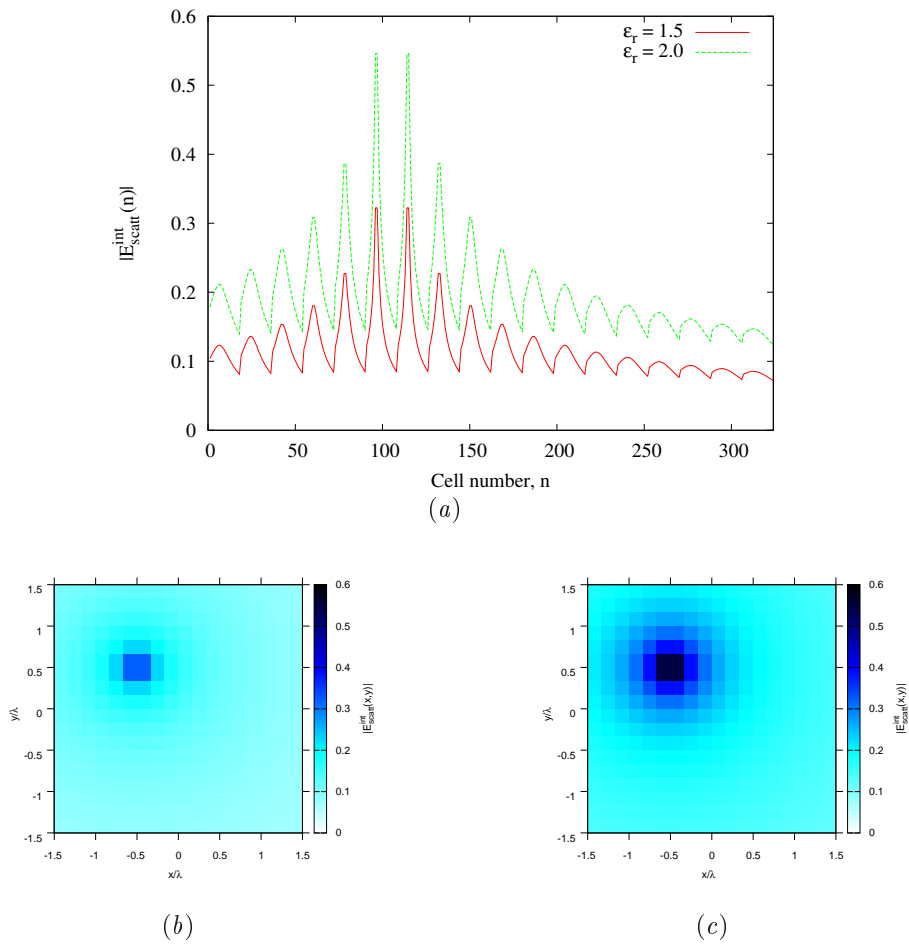
(b)



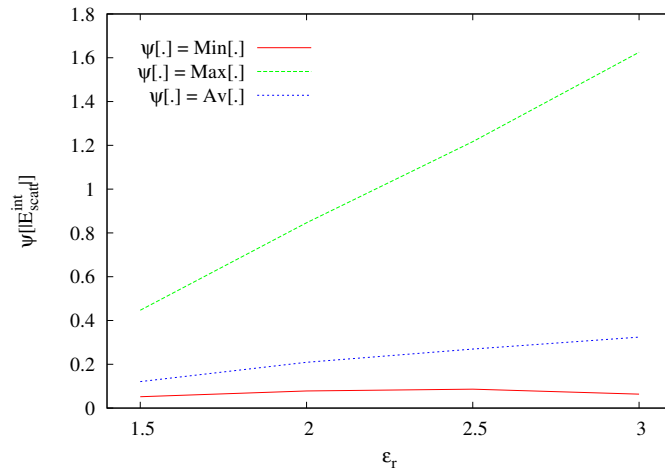
(c)

**Figure 14.** Behaviour of error figures as a function of  $\varepsilon_r$ , for different  $SNR$  values: (a) total error  $\xi_{tot}$ , (b) internal error  $\xi_{int}$ , (c) external error  $\xi_{ext}$ .

# Internal Scattered Field Analysis



**Figure 15.** (a) Average of the absolute value of the scattered field inside the investigation domain for different values of  $\epsilon_r$  : (b)  $\epsilon_r = 1.5$ , (c)  $\epsilon_r = 2.0$ .



(a)

**Figure 16.(a)** Internal Scattered Field statistical analysis.

**Observations:**

Le prestazioni della tecnica si possono considerare buone fino a  $\varepsilon_r = 2.0$ , ossia, osservando Fig.16.(a), anche in questo caso come nel TEST CASE precedentemente analizzato (singolo quadrato, dimensioni  $\lambda/6$ ), fino a quando  $\frac{E_{tot}^{int} - E_{inc}^{int}}{E_{inc}^{int}} < 0.2$ .



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